

**THE PROPAGATION OF SOUND  
OVER GROUND WITH AND WITHOUT  
ACOUSTIC BARRIERS**

**HANS JONASSON**





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## PREFACE

This thesis is submitted for the doctor's examination in Building Acoustics.

The work has been carried out at the Division of Building Technology, Lund Institute of Technology.

The author wishes to express his gratitude to Bengt Bengtsson, who with great skill managed all practical details concerning the measurements, and to Lena Fåke, who patiently typed the manuscript and drew the figures.

The author is also indebted to AB Lättbetong for being kind enough to erect the cellular concrete wall and to the authorities concerned for granting the use of a lawn for this wall.

Lund in May, 1971

Hans Jonasson

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## SUMMARY

An existing theory by Weyl & Ingard on the propagation of spherical sound waves over ground with a finite acoustic impedance is verified by full-scale measurements. This theory is combined with modern diffraction theory according to Clemmow & Senior to give a solution of the mixed problem with an acoustic barrier on the ground. It is shown that, with some approximations, this solution can be expressed in terms of the free field diffracted wave and some kind of spherical reflection coefficients which can be calculated from the theory of Weyl & Ingard. The validity of different free field diffraction theories is also investigated. An analysis of the nature of the diffracted wave leads to an approximate solution which is useful for low frequencies. The theoretical results are verified by a great number of full-scale measurements, which have been carried out with a 2.8 m high and 30 m wide wall on level grassland. Finally some practical applications to the propagation of traffic noise are given.





## LIST OF SYMBOLS

Unless otherwise stated, the symbols most commonly used have the following meaning:

|              |                                                                                     |
|--------------|-------------------------------------------------------------------------------------|
| A            | amplitude factor                                                                    |
| $C(x)$       | $\int_0^x \cos \frac{\pi}{2} t^2 dt$                                                |
| D            | horizontal distance between source and receiver                                     |
| $F(x)$       | $\int_0^x e^{it^2} dt$                                                              |
| G            | special function                                                                    |
| H            | Hankel function                                                                     |
| I            | integral function                                                                   |
| L,P          | paths of integration                                                                |
| Q            | reflection coefficient for spherical and cylindrical waves                          |
| R            | reflection coefficient for plane waves, direct distance between source and receiver |
| $R_1$        | distance between source and receiver via the top of the barrier                     |
| $S(x)$       | $\int_0^x \sin \frac{\pi}{2} t^2 dt$                                                |
| BH           | barrier height                                                                      |
| RH           | receiver's height above the ground                                                  |
| SH           | source's height above the ground                                                    |
| c            | velocity of sound                                                                   |
| g            | special function                                                                    |
| i            | $\sqrt{-1}$                                                                         |
| k            | wave number                                                                         |
| m            | $\cos \theta$                                                                       |
| n            | natural number                                                                      |
| p            | sound pressure                                                                      |
| r,s          | distance                                                                            |
| t,u          | variables of integration                                                            |
| x,y,z        | rectangular coordinates                                                             |
| $\dot{x}, y$ | variables                                                                           |



|                                 |                                          |
|---------------------------------|------------------------------------------|
| $\alpha, \beta, \gamma, \delta$ | angles                                   |
| $\zeta$                         | specific acoustic impedance              |
| $\eta$                          | angle                                    |
| $\theta$                        | angle of incidence on the ground surface |
| $\rho$                          | numerical distance                       |
| $\alpha$                        | diffraction parameter                    |
| $\tau$                          | variable of integration                  |
| $\upsilon$                      | specific acoustic admittance             |
| $\phi$                          | angle of diffraction                     |
| $\psi$                          | phase angle                              |
| $\omega$                        | angular frequency                        |

Indices are used as follows:

|      |                                               |
|------|-----------------------------------------------|
| $d$  | diffracted wave (upper index)                 |
| $R$  | reflected wave                                |
| $+1$ | reflected wave when $Q = +1$                  |
| $o$  | denotes quantities associated with the source |

In the text complex quantities are underlined.





## 1. INTRODUCTION

To be able to calculate the reduction of sound by barriers on the ground the effects that must be considered are reflections from the ground and diffraction by the barriers. Both effects have been treated separately before. The aim of this work has been to couple them together and thus obtain a theoretical solution of the mixed problem.

The propagation of a spherical sound wave over the ground was first solved approximately by WEYL 1919. By assuming that the ground surface was locally reacting and thus could be characterized by its acoustic point impedance INGARD 1951 achieved an exact solution by using WEYL's theories. RUDNICK 1947 solved the corresponding problem with more complicated impedance conditions for the ground. His work was based on theories by NORTON 1937.

The above theories have so far not been verified on any larger scale by field measurements. TILLOTSON 1966 found good correlation between Rudnick's theory and experimental values on the attenuation of sound over snowcovered fields and ONCLEY 1970 reported some qualitative agreement between theory and field measurements of jet engine noise over sandy soil. Laboratory measurements on an acoustic wave propagated along a sound absorbing boundary were performed by LAWHEAD & RUDNICK 1951 and DELANY & BAZLEY 1970 and the reported results were in excellent agreement with both Ingard's and Rudnick's theories. Very extensive series of measurements of the horizontal propagation of jet engine noise over level grassland were carried out by PARKIN & SCHOLLES 1964-65 and SCHOLLES & PARKIN 1967 but no attempts were made to explain the results theoretically.

The first exact solution of a diffraction problem was the one by SOMMERFELD 1896, who treated the diffraction of a plane wave by a half-plane. The corresponding problem for spherical waves was exactly solved by MACDONALD 1915. This solution seems to be virtually unknown to acousticians. Instead they use the classical Kirchhoff theory or the solution by REDFERN 1940, which however are both good approximations.

The attempts to take ground reflections into account have been few. FEHR 1951 put both source and receiver in a totally reflecting plane and thus decreased the free field attenuation by 6 dB. In this way he did not get





the absurd "free field attenuation" of 6 dB with an infinitely low barrier on the ground. LUKASIK & NOLLE 1955 gave a correct qualitative explanation of the effect of the ground reflections on the reduction of sound by a barrier but unfortunately few people seem to have read and understood their report. MAEKAWA 1965 treated the case with both source and receiver on a finite height over a totally reflecting ground surface. He also gave the solution with different reflection coefficients without indicating how to compute them. LINDBLAD 1968 was the first one to take the finite impedance of the ground into account. He applied Huygens' principle to both diffraction and propagation over the ground. In those cases when the spherical reflection coefficient approaches the plane one his solution might be a useful approximation.

All field measurements carried out on barriers up to the present are at least from a theoretical point of view of less value because they lack information about the ground impedance. Besides they often lack a useful reference level. Measured values have generally been compared with one or both of the free field and Fehr's totally reflecting diffraction theory and the correlation between theory and experiment has as a rule been unsatisfactory. JENSEN 1969 and FLEISCHER 1970 are examples of measurements published lately.

In this paper the propagation over ground is treated by Ingard's theory which is also shown to be valid for Rudnick's impedance conditions in most practical cases. This theory is then combined with the diffraction theory that was developed by CLEMMOW 1950-51 and later applied on the half-plane problem for spherical waves by SENIOR 1953. As the theories have had to be slightly rearranged to suit the purposes of this work they have been recapitulated to a large extent.

The only type of barrier considered in this paper is the infinitely thin one. This restriction is of minor importance as the barrier shape comparatively insignificantly influences the sound reduction by the barrier. This has among other things been verified by model measurements carried out by RINGHEIM 1971. For more theoretical details about the diffraction by different barrier shapes the book by BOWMAN, SENIOR & USLENGHI 1969 is recommended. The barrier is further assumed to be of infinite width and only the geometrical shadow zone is considered. The necessary extensions that are required to cover the other



cases are self-evident.

All effects of turbulence, wind- and temperature-gradients have been neglected throughout. This does not mean that they are of minor importance. They are however beyond the scope of this work.

The solution of the mixed ground-barrier problem is obtained by superposition of four different wave-types. These are treated one at a time in Sections 2.3 and 2.4. The complete solution is finally given in Section 2.5.

The comparison between solutions of the diffraction of a spherical sound wave by a half-plane has been included because they are often quoted as completely different solutions. As is shown in Section 2.3 they are in general quite equivalent.

The propagation of cylindrical waves over the ground has been included because as is shown in Section 2.3.2,4 the diffracted wave in some cases behaves like a cylindrical one.

Those of the readers who dislike mathematics are advised to skip the theoretical analysis in Section 2. References are given to the most important equations in this section from Sections 3 and 4.





## 2. THEORETICAL ANALYSIS

The time-dependence  $e^{-i\omega t}$  is assumed and it is omitted throughout. All complex quantities are underlined.

### 2.1. The acoustic properties of the ground

An extensive treatment of the acoustic fundamentals underlying this section is found in MORSE-INGARD 1968, Chapter 6.

#### 2.1.1. THE PLANE WAVE REFLECTION COEFFICIENT

A boundary surface responding to an incident pressure wave may either be one of local or extended reaction. In the former case one point of the surface is moving independent of any other point of the surface and this one can then be characterized by its specific (point-) impedance  $\underline{\zeta}$ . The pressure reflection coefficient for a plane wave incident on the

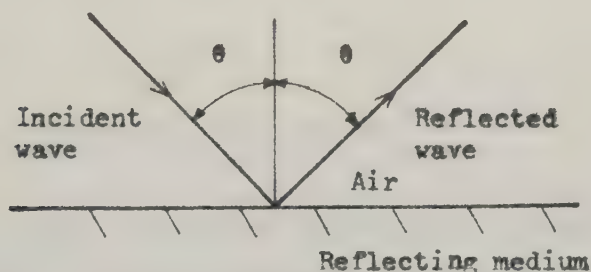


FIG. 2.1.  
Reflection from a  
plane surface

boundary surface as shown in Fig. 2.1 is then

$$\underline{R} = \frac{\underline{\zeta} \cos \theta - 1}{\underline{\zeta} \cos \theta + 1} \quad (2.1)$$

In the case of extended reaction we have to take the wave propagation within the boundary medium into account. The medium can then be described by its specific characteristic impedance  $\underline{\zeta}_c$  and wave propagation velocity  $\underline{c}_m$ . The reflection coefficient will then be given by

$$\underline{R} = \frac{\underline{\zeta}_c \cos \theta - \{1 - (\underline{c}_m/c)^2 \sin^2 \theta\}^{1/2}}{\underline{\zeta}_c \cos \theta + \{1 - (\underline{c}_m/c)^2 \sin^2 \theta\}^{1/2}} \quad (2.2)$$

where  $c$  is the velocity of sound in air. By introducing an effective specific impedance



$$\underline{\zeta}_e = \underline{\zeta}_c \{1 - (\underline{c}_m/c)^2 \sin^2 \theta\}^{-1/2} \quad (2.3)$$

Eq. (2.2) can be written in the same form as Eq. (2.1). When  $|\underline{c}_m| \ll c$   $\underline{\zeta}_e$  will no longer be a function of  $\theta$  and thus the boundary surface will be one of local reaction.

In most practical applications we are interested in sound propagation near grazing incidence. Then  $\sin \theta \approx 1$  and the  $\theta$ -dependence of  $\underline{\zeta}_e$  is negligible, that is  $\underline{\zeta}_e$  can be treated like a point impedance. By redefining  $\underline{\zeta}$  to cover this special case Eq. (2.1) will be valid for both kinds of boundary surfaces. In the following this extended equation will be used in the form

$$\underline{R} = \frac{\cos \theta - \underline{v}}{\cos \theta + \underline{v}} \quad (2.4)$$

where the effective point admittance

$$\underline{v} = \frac{1}{\underline{\zeta}} \quad (2.5)$$

### 2.1.2. MEASUREMENT OF IMPEDANCE

If the ground surface is uniform and uncovered by vegetation conventional impedance tube measurements are accomplishable. As was shown by FERRERO & SACERDOTE 1951 it is then possible to determine both  $\underline{\zeta}_c$  and  $\underline{c}_m$  by measuring on two different thicknesses of the test material. For more complicated ground surfaces it is however necessary to carry out the measurements in the field.

DICKINSON 1969 and DICKINSON & DOAK 1970 used both the impedance tube method and a new spherical free field standing wave method in the field. Both methods presuppose that the ground surface is one of local reaction. Dickinson found that the impedance tube worked badly because it changed the climate within the tube and he thus recommends the free field method. The accuracy of this method is, however, less satisfactory when the ground is acoustically hard. In that case the phase shift of the reflected wave is very small and to detect this shift the distance from the first sound pressure minimum to the ground level must at times be known with an accuracy of about a mm. This is hardly realistic. On acoustically soft





surfaces the method might however be useful.

A sound pressure gradient method suggested by MECHEL 1968 has got the same limitations as Dickinson's free field method.

By using the theories of Section 2.2.1 the effective admittance  $\underline{u}$  of Eq. (2.4) can be determined indirectly by measuring the sound pressure distribution due to a point source close to the ground. Mathematically this method is less satisfactory because trial and error solution methods have to be applied but otherwise it has got certain advantages. It does not matter whether the ground surface is locally reacting or not and small local inhomogeneities will not appreciably affect the result. As the measurement sites turned out to be acoustically very hard, that is  $|\underline{u}|$  was small, this indirect method was the only one that was successful. Thus all computations in Section 3 have been performed with admittance data derived from propagation measurements.

## 2.2. The propagation of sound over the ground

Suitable background references for this section are BRUIJN 1961, Chapter 5, COURANT & HILBERT 1962, Chapter III, § 3, MORSE & FESHBACH 1953, Chapter 4 and SOMMERFELD 1966 §§ 19 and 21.

In Section 2.2.1 the analysis of INGARD 1951 is repeated because it is essential to the understanding of the following. The same methods are applied to cylindrical waves. This has not been done before although the analysis is all but self-evident when the spherical solution is known. Cylindrical waves are studied because diffracted waves sometimes behave like cylindrical ones.

### 2.2.1. SPHERICAL WAVES

The sound pressure produced by a point source can be expressed in an integral form as

$$\underline{p} = \frac{e^{ikr}}{kr} = \frac{i}{2\pi} \int_0^{2\pi} \int_L e^{ik(ax + by + cz)} \sin \theta \, d\theta d\phi \quad (2.6)$$



where  $a$ ,  $b$  and  $c$  are the direction cosines of the propagation vector and  $L$  the path of integration in the complex  $\underline{b}$ -plane as shown in Fig. 2.2.

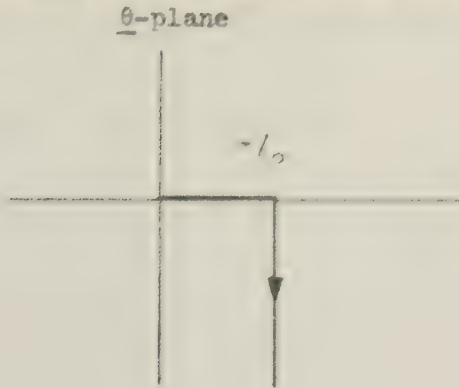


FIG. 2.2.  
The path  $L$  in the  $\underline{b}$ -plane.

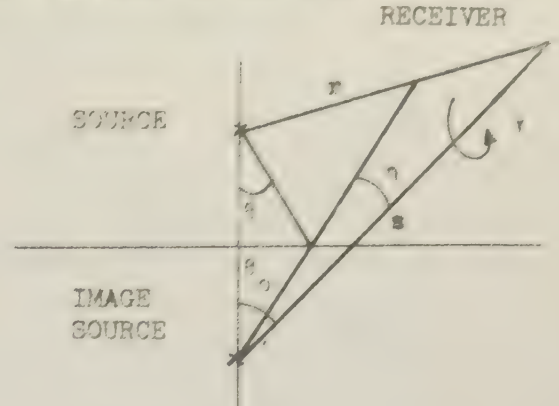


FIG. 2.3.  
Geometry of sound source and receiver.

The integrand of Eq. (2.6) is a plane wave and by multiplying by the plane wave reflection coefficient as defined by Eq. (2.4) and moving the original source to the image source position we will get the reflected wave. With new polar coordinates around the axis image source-receiver as shown in Fig. 2.3 the reflected wave will thus be given by

$$p_R = \frac{i}{2\pi} \int_0^{2\pi} \int_L e^{iks \cos \underline{\eta}} \underline{R}(\gamma, \underline{\eta}) \sin \underline{\eta} \, d\gamma d\underline{\eta} \quad (2.7)$$

The path of integration must be the same as before.  $\underline{R}(\gamma, \underline{\eta})$  is given from

$$\underline{R}(\gamma, \underline{\eta}) = \frac{\cos \underline{\theta} - \underline{v}}{\cos \underline{\theta} + \underline{v}} \quad (2.8)$$

where

$$\cos \underline{\theta} = \cos \theta_0 \cos \underline{\eta} + \sin \theta_0 \sin \underline{\eta} \cos \gamma \quad (2.9)$$

The steepest descent substitution

$$\cos \underline{\eta} = 1 + i t \quad (2.10)$$

transforms Eq. (2.7) into

$$p_R = \frac{e^{iks}}{ks} \int_0^{\infty} e^{-kst} \underline{R}(t) \, dt = \underline{Q} \frac{e^{iks}}{ks} \quad (2.11)$$





where  $\underline{Q}$  is some kind of spherical reflection coefficient and

$$\underline{R}(t) = \frac{1}{2\pi} \int_0^{2\pi} \underline{R}(t, \gamma) d\gamma \quad (2.12)$$

This integral is of the type

$$\int_0^{2\pi} \frac{d\gamma}{1 + \underline{A} \cos \gamma} = 2\pi(1 - \underline{A}^2)^{-1/2} \quad (2.13)$$

and thus we finally get

$$\underline{R}(t) = 1 - 2\underline{v}\{(m + \underline{v})^2 + 2it(1 + \underline{v}m) - t^2\}^{-1/2} \quad (2.14)$$

where

$$m = \cos \theta_0 \quad (2.15)$$

As the greatest contribution to the integral in Eq. (2.11) comes from the neighbourhood of the saddlepoint  $t = 0$  we may neglect  $t^2$  in Eq. (2.14).  $\underline{Q}$  may now be written in the form

$$\underline{Q} = \underline{R}_0 + (1 - \underline{R}_0) \underline{G}(\underline{\rho}) \quad (2.16)$$

where

$$\underline{R}_0 = \frac{m - \underline{v}}{m + \underline{v}} \quad (2.17)$$

$$\underline{G}(\underline{\rho}) = 1 - \underline{g}(\underline{\rho}) \quad (2.18)$$

$$\underline{g}(\underline{\rho}) = \int_0^{\infty} e^{-t} (1 - t/\underline{\rho})^{-1/2} dt \quad (2.19)$$

$$\underline{\rho} = iks \frac{(m + \underline{v})^2}{2(1 + m\underline{v})} \quad (2.20)$$

It should be observed that Ingard defines his numerical distance  $\underline{\rho}$  with a minus-sign and that this sign has been lost in his paper.



When  $\underline{\rho}$  is reasonably small the asymptotic series

$$\begin{aligned}\underline{G}(\underline{\rho}) &= 1 - ie^{-\underline{\rho}} \sqrt{\pi \underline{\rho}} (1 - 2i\sqrt{\underline{\rho}}/\sqrt{\pi} \sum_{n=0}^{\infty} \frac{\underline{\rho}^n}{n!(2n+1)}) = \\ &= 1 - ie^{-\underline{\rho}} \sqrt{\pi \underline{\rho}} - 2e^{-\underline{\rho}} \underline{\rho} (1 + \frac{\underline{\rho}}{1!3} + \frac{\underline{\rho}^2}{2!5} + \frac{\underline{\rho}^3}{3!7} + \dots) \\ &\quad (2.21)\end{aligned}$$

is useful for computations. For large values of  $\underline{\rho}$  the semi-convergent series

$$\underline{G}(\underline{\rho}) = - \sum_{n=1}^{\infty} \frac{(2n)!}{2^n n! (2\underline{\rho})^n} = -(\frac{1}{2\underline{\rho}} + \frac{1 \cdot 3}{(2\underline{\rho})^2} + \frac{1 \cdot 3 \cdot 5}{(2\underline{\rho})^3} + \dots) \quad (2.22)$$

is more convenient

The phase angle of  $\underline{\rho}$  extends from  $-3\pi/2$  to  $\pi/2$ . As the function  $\underline{G}$  has its branch cut along the positive real axis the discontinuity occurring here must be moved to the positive imaginary axis. This means that Eq. (2.21) is not valid when  $\underline{\rho}$  is in the first quadrant of the complex plane. If the  $\underline{G}$ -function in this case is denoted  $\underline{G}_1$  we get

$$\underline{G}_1(\underline{\rho}) = |\underline{G}(\overline{\underline{\rho}})| \exp\{i(2\psi(|\underline{\rho}|) - \psi(\overline{\underline{\rho}}))\} \quad (2.23)$$

where  $\overline{\underline{\rho}}$  denotes the complex conjugate of  $\underline{\rho}$  and  $\psi(\underline{\rho})$  the phase angle of  $\underline{G}(\underline{\rho})$ . In the following the function  $\underline{G}(\underline{\rho})$  will be defined as  $\underline{G}_1(\underline{\rho})$  in the first quadrant of the  $\underline{\rho}$ -plane.

The total field at the receiver is given by superposition of the direct and the reflected wave. Thus

$$\underline{p} = \frac{e^{ikr}}{kr} (1 + \underline{Q} \frac{r}{s} e^{ik(s-r)}) \quad (2.24)$$

For the computations in Section 3 the Eqs. (2.16), (2.17) and (2.20) - (2.24) have been used.





## 2.2.2. CYLINDRICAL WAVES

The geometry of source and receiver is the same as in Fig. 2.3.

The sound pressure produced by a line source is given by

$$\underline{p} = \sqrt{\frac{\pi}{2}} e^{i\pi/4} \underline{H}_0^{(1)}(kr) \quad (2.25)$$

By inserting Sommerfeld's integral representation of the Hankel-function

$$\underline{H}_0^{(1)}(kr) = \frac{2}{\pi} \int_L e^{ikr \cos \underline{\theta}} d\underline{\theta} \quad (2.26)$$

where  $L$  is the same path as in Fig. 2.2 and making the substitution  $\cos \underline{\theta} = 1 + it^2$  we obtain

$$\underline{p} = \frac{e^{ikr}}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-krt^2} (1 + it^2/2)^{-1/2} dt \quad (2.27)$$

At about a wave-length or more from the source a good approximation is

$$\underline{p} = \frac{e^{ikr}}{\sqrt{kr}} \quad (2.28)$$

Performing the same steps as in Section 2.2.1 and using Eqs. (2.4), (2.25) and (2.26) yield the reflected wave

$$\underline{p}_R = \frac{2e^{i\pi/4}}{\sqrt{2\pi}} \int_L e^{iks \cos \underline{\eta}} \left(1 - \frac{2\underline{v}}{\cos \underline{\theta} + \underline{v}}\right) d\underline{\eta} \quad (2.29)$$

where

$$\cos \underline{\theta} = \cos \theta_0 \cos \underline{\eta} + \sin \theta_0 \sin \underline{\eta} \quad (2.30)$$

The first integral will just yield a wave reflected with the reflection coefficient +1. After the substitution  $\cos \underline{\eta} = 1 + it^2$  the other integral will be

$$\underline{I}(\theta_0) = \frac{4\underline{v} e^{i\pi/4} e^{iks}}{\sqrt{2\pi}} \int_0^{\infty} \frac{e^{-kst^2} 2idt}{\{m + imt^2 + \{(1-m^2)(t^4 - 2it^2)\}^{1/2} + \underline{v}\} \{t^2 - 2i\}^{1/2}} \quad (2.31)$$



where  $m = \cos \theta_0$ . When  $ks$  is reasonably large the significant contributions to the integral come from the neighbourhood of  $t = 0$  and thus  $t^2$  can be neglected in the denominator. Hence

$$\begin{aligned} \underline{I}(\theta_0) &= \frac{8i\underline{u} e^{i\pi/4} e^{iks}}{\sqrt{2\pi}} \int_0^\infty \frac{e^{-kst^2}}{\sqrt{-2i} \{m + \underline{u} + \sqrt{-2i} \sqrt{1 - m^2 t}\}} dt = \\ &= - \frac{4\underline{u} e^{iks} e^{i\pi/4}}{\sqrt{2\pi} \sqrt{1 - m^2}} \int_0^\infty \frac{e^{-u^2}}{u + \underline{\rho}} du = \frac{e^{iks}}{\sqrt{ks}} (\underline{R}_0 - 1) \underline{g}(\underline{\rho}) \end{aligned} \quad (2.32)$$

Where  $\underline{R}_0$  is the plane reflection coefficient for the angle  $\theta_0$  and

$$\underline{g}(\underline{\rho}) = \frac{2\underline{\rho}}{\sqrt{\pi}} \int_0^\infty \frac{e^{-u^2}}{u + \underline{\rho}} du \quad (2.33)$$

$$\underline{\rho}^2 = iks \frac{(m + \underline{u})^2}{2(1 - m^2)} \quad (2.34)$$

Adding the +1-reflected wave to Eq. (2.32) gives the cylindrical reflection factor

$$\underline{Q} = \underline{R}_0 + (1 - \underline{R}_0) \underline{G}(\underline{\rho}) \quad (2.35)$$

where

$$\underline{G}(\underline{\rho}) = 1 - \underline{g}(\underline{\rho}) \quad (2.36)$$

The integral in Eq. (2.33) is well known although it cannot be solved in a closed form. An asymptotic series expansion of  $\underline{G}(\underline{\rho})$  that is convenient for reasonably small  $\underline{\rho}$  is

$$\begin{aligned} \underline{G}(\underline{\rho}) &= 1 + \frac{2\underline{\rho}}{\sqrt{\pi}} \exp(-\underline{\rho}^2) \left\{ \ln \underline{\rho} - \sqrt{\pi} \sum_{n=0}^\infty \frac{2n+1}{n!(2n+1)} - \right. \\ &\quad \left. - \sum_{n=1}^\infty \frac{\underline{\rho}^{2n}}{n! 2n} - \frac{\gamma}{2} \right\} \end{aligned} \quad (2.37)$$

where  $\gamma = \text{Euler's constant} = 0.5772\dots\dots$ . A semi-convergent series that is more convenient for large  $\underline{\rho}$  is





$$\begin{aligned}
\underline{G}(\underline{\rho}) &= -\frac{2}{\sqrt{\pi}} \left\{ \frac{\sqrt{\pi}}{4} \left( \frac{1}{2} + \frac{1 \cdot 3}{\underline{\rho}} + \dots \dots \frac{1 \cdot 3 \dots \dots (2n-1)}{\underline{\rho}^{2n}} \right) - \right. \\
&\quad \left. - \frac{1}{2\underline{\rho}} \left( 1 + \frac{1!}{2} + \frac{2!}{4} + \dots \dots \frac{n!}{2n} \right) \right\} = \\
&= \frac{1}{\sqrt{\pi}} \cdot \frac{1}{\underline{\rho}} - \frac{1}{2} \cdot \frac{1}{\underline{\rho}} + \frac{1}{\sqrt{\pi}} \cdot \frac{1}{\underline{\rho}^3} - \frac{3}{2} \cdot \frac{1}{\underline{\rho}^4} + O(\underline{\rho}^{-5}) \quad (2.38)
\end{aligned}$$

As for spherical waves the total field at the receiver is given by superposition. We thus get

$$\underline{p} = \frac{e^{ikr}}{\sqrt{kr}} \left( 1 + \underline{Q} \sqrt{\frac{r}{s}} e^{ik(s-r)} \right) \quad (2.39)$$

Eq. (2.28) has been used to get the direct field as the approximation that was made between Eqs. (2.27) and (2.28) is equivalent to the one between Eqs. (2.31) and (2.32).

For computations Eqs. (2.4), (2.34), (2.35) and (2.37) - (2.39) are required.

### 2.3. The diffraction of sound by a half-plane

A useful theoretical basis for this section is Section 2.2 and SOMMERFELD 1964, Chapter 5. For an extensive review of different diffraction theories BOUWKAMP 1954 is recommended.

The plane wave solution in Section 2.3.1 has been included because it is used when deriving the solution for spherical waves. Besides some special functions which frequently are used in diffraction theory are introduced in that section.

#### 2.3.1. PLANE WAVES

A detailed derivation of the solution of this problem can be found in SOMMERFELD 1964. Only the results will be reproduced below.



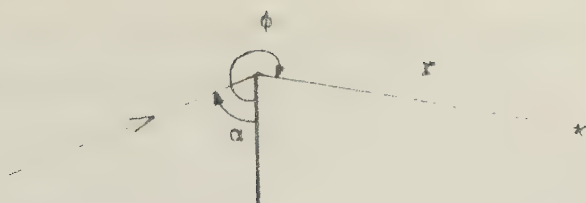


FIG. 2.4.  
Geometry of half-plane  
with incident and  
diffracted plane wave.

If the incident plane wave is propagating in the direction  $\phi = \alpha + \pi$  that is

$$p_{\cdot}^{\text{in}} = e^{ikr \cos(\phi - \alpha)} \quad (0 < \alpha < \pi) \quad (2.40)$$

the solution for the diffracted field ( $\phi > \alpha + \pi$ ) can be written in terms of the Fresnel integral as

$$p_{\cdot}^{\text{d}} = \frac{e^{-i\pi/4}}{\sqrt{\pi}} \left\{ e^{-ikr \cos(\phi - \alpha)} \underline{F}(-\sqrt{2kr} \cos \frac{\phi - \alpha}{2}) + \right. \\ \left. + e^{-ikr \cos(\phi + \alpha)} \underline{F}(-\sqrt{2kr} \cos \frac{\phi + \alpha}{2}) \right\} \quad (2.41)$$

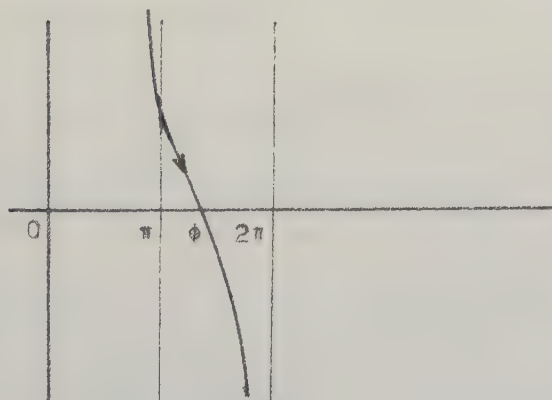


FIG. 2.5.  
The path  $P(\phi)$  in the  $\underline{\gamma}$ -plane.

or in the integral form as

$$p_{\cdot}^{\text{d}} = \frac{-i}{4\pi} \int_{P(\phi)} \left\{ \frac{1}{\cos(\underline{\gamma} - \alpha)/2} + \frac{1}{\cos(\underline{\gamma} + \alpha)/2} \right\} e^{ikr \cos(\underline{\gamma} - \phi)} d\underline{\gamma} \quad (2.42)$$

where the plus-sign is valid for a totally reflecting and the minus-sign for a totally absorbing half-plane.  $P(\phi)$  denotes the path of integration





in the  $\gamma$ -plane which is shown in Fig. 2.5. The definition of the Fresnel integral in Eq. (2.41) is

$$\underline{F}(x) = \int_x^{\infty} e^{it^2} dt \quad (2.43)$$

The asymptotic expansion for  $\underline{F}(x)$  is at infinity

$$\underline{F}(x) = e^{ix^2} \frac{i}{2x} \left( 1 + \frac{1}{2ix^2} + \frac{1 \cdot 3}{(2ix^2)^2} + \frac{1 \cdot 3 \cdot 5}{(2ix^2)^3} + \dots \right) \quad (2.44)$$

and at zero

$$\underline{F}(x) = \frac{\sqrt{\pi}}{2} e^{i\pi/4} - x \left( 1 + \frac{i}{1 \cdot 3} x^2 - \frac{1}{2 \cdot 5} x^4 - \frac{i}{3 \cdot 7} x^6 + \dots \right) \quad (2.45)$$

In many practical cases  $\phi + \alpha$  is of the order of magnitude  $2\pi$  and  $kr$  is large. Thus

$$\underline{F}(-\sqrt{2kr} \cos \frac{\phi + \alpha}{2}) \approx 0 \quad (2.46)$$

Eq. (2.41) now simplifies to

$$\underline{p}^d = \frac{e^{-i\pi/4}}{\sqrt{\pi}} e^{-ikr \cos(\phi - \alpha)} \underline{F}(\sigma) \quad (2.47)$$

where

$$\sigma = -\sqrt{2kr} \cos \frac{\phi - \alpha}{2} \quad (2.48)$$

This means that the character of the half-plane surface is irrelevant. Presupposing that  $\sigma$  is large we get using the first term of the asymptotic expansion in Eq. (2.44)

$$\underline{p}^d = \frac{e^{-i 3\pi/4}}{2 \sqrt{2\pi}} \frac{1}{\cos(\phi - \alpha)/2} \frac{e^{ikr}}{\sqrt{kr}} \quad (2.49)$$

From this expression we can see that for a given  $\phi$  the diffracted wave behaves like a cylindrical one.



Eq. (2.47) is often written in the form

$$p^d = \frac{e^{-i\pi/4}}{\sqrt{2}} e^{-ikr \cos(\phi-\alpha)} \left\{ \frac{1}{2} - C\left(\sqrt{\frac{2}{\pi}} \sigma\right) + i\left(\frac{1}{2} - S\left(\sqrt{\frac{2}{\pi}} \sigma\right)\right) \right\} \quad (2.50)$$

where

$$C(x) = \int_0^x \cos \frac{\pi}{2} t^2 dt \quad (2.51)$$

$$S(x) = \int_0^x \sin \frac{\pi}{2} t^2 dt \quad (2.52)$$

### 2.3.2. SPHERICAL WAVES

In the first subsection an approximate solution is derived using the method of SENIOR 1953. His exact solution has had to be slightly modified and approximated in order to make it useful for the corresponding problem for the reflected wave that will be solved in Section 2.3.3. The validity of performed approximations will be evident from the comparisons of different solutions that follow in Subsection 2.3.2.3.

The nature of the diffracted wave is studied because it is essential for the understanding of the propagation conditions over ground when there is a sound barrier present.

#### 2.3.2.1. An approximate solution

By expressing the spherical wave as a double integral over plane waves we can make use of Sommerfeld's solution of the corresponding problem for plane waves.

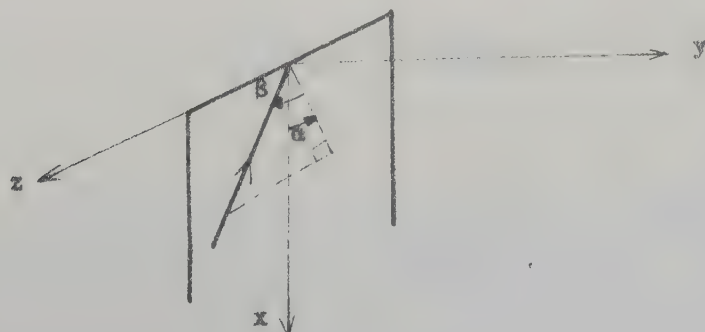


FIG. 2.6.  
Geometry of half-plane  
with incident spherical  
wave.





With circular cylindrical coordinates a spherical scalar wave can be expressed as

$$\underline{p} = -\frac{i}{2\pi} \int_0^{2\pi} d\alpha \int_{\frac{\pi}{2}}^{\pi-i\infty} e^{ikr \cos(\phi-\alpha)\cos\beta+ikz \sin\beta \cos\beta} d\beta \quad (2.53)$$

Splitting up the  $\alpha$ -integral and changing the  $\beta$ -path gives

$$\begin{aligned} \underline{p} = & -\frac{i}{2\pi} \int_0^{\pi} d\alpha \int_{+i\infty}^{\frac{\pi}{2}} e^{-ikr \cos(\phi-\alpha)\cos\beta+ikz \sin\beta \cos\beta} d\beta - \\ & -\frac{i}{2\pi} \int_0^{\pi} d\alpha \int_{\frac{\pi}{2}}^{\pi-i\infty} e^{-ikr \cos(\phi-\alpha) \cos\beta+ikz \sin\beta \cos\beta} d\beta \end{aligned} \quad (2.54)$$

Adding these two integrals and introducing a phase factor due to the fact that the source is not situated at the origin but at the point  $(r_0 \cos \phi_0, r_0 \sin \phi_0, z_0)$  we get the incident wave

$$\begin{aligned} \underline{p}^{\text{in}} = & -\frac{i}{2\pi} \int_0^{\pi} d\alpha \int_{P(\frac{\pi}{2})} e^{-ik \{r \cos(\phi-\alpha) \cos\beta - r_0 \cos(\phi_0-\alpha) \cos\beta\} +} \\ & + ik(z-z_0) \sin\beta \cos\beta} d\beta \end{aligned} \quad (2.55)$$

where the steepest descent path  $P$  corresponds to the one in Fig. 2.5. The integrand is now a threedimensional plane wave and we may make use of the solution (2.42) if we generalize it to three dimensions. Eq. (2.42) then transforms into

$$\underline{p}_P^{\text{d}} = \frac{-i}{4\pi} \int_{P(\phi)} \left\{ \frac{1}{\cos(\gamma-\alpha)/2} + \frac{1}{\cos(\gamma+\alpha)/2} \right\} e^{ikr \cos\beta \cos(\gamma-\phi)+ikz \sin\beta} d\gamma \quad (2.56)$$

where the index  $P$  denotes plane waves. The diffracted spherical wave is now given by the last two equations. Hence



$$\begin{aligned}
 p^d = & -\frac{1}{8\pi^2} \int_0^\pi \int_{P(\frac{\pi}{2})} \int_{P(\phi)} e^{ikr \cos(\underline{\gamma}-\phi) \cos\beta + ikr_0 \cos(\phi_0-\alpha) +} \\
 & +ik(z-z_0)\sin\beta \frac{1}{\cos(\underline{\gamma}-\alpha)/2} + \frac{1}{\cos(\underline{\gamma}+\alpha)/2} \cos\beta \, d\alpha d\beta d\underline{\gamma}
 \end{aligned}
 \tag{2.57}$$

Changing the  $\underline{\gamma}$ -path to go through  $\underline{\gamma}=0$  and changing the sign of  $\underline{\gamma}$  in the first integral gives

$$\begin{aligned}
 p^d = & -\frac{1}{8\pi^2} \int_0^\pi \int_{P(\frac{\pi}{2})} \int_{P(0)} e^{ikr \cos\underline{\gamma} \cos\beta + ikr_0 \cos(\phi_0-\alpha) + ik(z-z_0)\sin\beta} \, x \\
 & x \left\{ \frac{1}{\cos(\underline{\gamma}+\alpha-\phi)/2} + \frac{1}{\cos(\underline{\gamma}+\alpha+\phi)/2} \right\} \cos\beta \, d\alpha d\beta d\underline{\gamma}
 \end{aligned}
 \tag{2.58}$$

Introducing the new variable  $\underline{\gamma}+\alpha=\underline{\delta}$  gives

$$\begin{aligned}
 p^d = & -\frac{1}{8\pi^2} \int_0^\pi \int_{P(\frac{\pi}{2})} \int_{P(\frac{\pi}{2})} e^{ikr \cos(\underline{\delta}-\alpha) \cos\beta + ikr_0 \cos(\phi_0-\alpha) + ik(z-z_0)\sin\beta} \, x \\
 & x \left\{ \frac{1}{\cos(\underline{\delta}-\phi)/2} + \frac{1}{\cos(\underline{\delta}+\phi)/2} \right\} \cos\beta \, d\alpha d\beta d\underline{\delta}
 \end{aligned}
 \tag{2.59}$$

where the  $\underline{\delta}$ -path has been distorted to go through the point  $\frac{\pi}{2}$  on the real axis. No residues will contribute to the integral as long as we just study the shadow-zone.

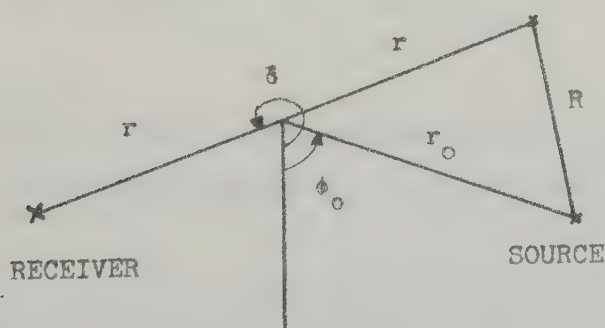


FIG. 2.7.  
Geometry of source  
and receiver.

From Eq. (2.55) we can see that the  $\alpha$ ,  $\underline{\beta}$ -integral in Eq. (2.59) may be integrated to

$$2\pi i \frac{e^{ikR}}{kR}$$



where  $R$  is the distance shown in Fig. 2.7. As  $\underline{\delta}$  is complex  $R$  becomes complex. Geometrical considerations give

$$\underline{R}^2 = r^2 + r_o^2 + 2rr_o \cos(\underline{\delta} - \phi_o) + (z - z_o)^2 \quad (2.60)$$

Eq. (2.59) now takes the form

$$\underline{p}^d = \frac{1}{4\pi i} \int_{P(\frac{\pi}{2})} \frac{e^{ik\underline{R}}}{k\underline{R}} \left\{ \frac{1}{\cos(\underline{\delta} - \phi)/2} + \frac{1}{\cos(\underline{\delta} + \phi)/2} \right\} d\underline{\delta} \quad (2.61)$$

The  $\underline{\delta}$ -path may be distorted into  $P(\phi_o)$  and the substitution  $\underline{\delta} = \underline{\delta}' + \phi_o$  finally gives

$$\underline{p}^d = \frac{1}{4\pi i} \int_{P(0)} \frac{e^{ik\underline{R}}}{k\underline{R}} \left\{ \frac{1}{\cos(\underline{\delta}' + \phi_o - \phi)/2} + \frac{1}{\cos(\underline{\delta}' + \phi_o + \phi)/2} \right\} d\underline{\delta}' \quad (2.62)$$

$$\text{where } \underline{R}^2 = r^2 + r_o^2 + 2rr_o \cos \underline{\delta}' + (z - z_o)^2 \quad (2.63)$$

By changing the sign of  $\underline{\delta}'$  in the first integral and dropping the prime-sign Eq. (2.62) can be written as

$$\underline{p}^d = \underline{I}(-\phi_o) + \underline{I}(\phi_o) \quad (2.64)$$

where

$$\underline{I}(-\phi_o) = \frac{1}{4\pi i} \int_{P(0)} \frac{e^{ik\underline{R}}}{k\underline{R}} \frac{d\underline{\delta}}{\cos(\underline{\delta} + \phi - \phi_o)/2} \quad (2.65)$$

Now multiply both nominator and denominator in Eq. (2.65) by  $\cos \frac{1}{2}(\underline{\delta} - \phi + \phi_o)$  and cancel odd functions of  $\underline{\delta}$ . Eq. (2.65) then takes the form

$$\underline{I}(-\phi_o) = \frac{1}{2\pi i} \int_{P(0)} \frac{e^{ik\{r^2 + r_o^2 + 2rr_o \cos \underline{\delta} + (z - z_o)^2\}^{1/2}} \cos \frac{\underline{\delta}}{2} \cos \frac{\phi - \phi_o}{2}}{k\{r^2 + r_o^2 + 2rr_o \cos \underline{\delta} + (z - z_o)^2\}^{1/2} \cos \underline{\delta} + \cos(\phi - \phi_o)} d\underline{\delta} \quad (2.66)$$

The steepest descent substitution  $\cos \underline{\delta} = 1 + i\tau^2$  now yields





$$\underline{I}(-\phi_0) = \frac{e^{-i\pi/4}}{\sqrt{2\pi i}} \int_{-\infty}^{\infty} \frac{\exp\{ik(R_1^2 + 2irr_0\tau^2)^{1/2}\}}{k\{R_1^2 + 2irr_0\tau^2\}^{1/2}} \frac{\cos(\phi - \phi_0)/2}{1 + i\tau^2 + \cos(\phi - \phi_0)} d\tau \quad (2.67)$$

where

$$R_1^2 = (r+r_0)^2 + (z-z_0)^2 \quad (2.68)$$

Approximating the exponent with the first three terms of a Maclaurin series and neglecting  $2irr_0\tau^2$  compared to  $R_1^2$  in the denominator yields

$$\underline{I}(-\phi_0) = \frac{e^{-i\pi/4}}{\sqrt{2\pi i}} \frac{\cos(\phi - \phi_0)/2}{kR_1} e^{ikR_1} \int_{-\infty}^{\infty} \frac{\exp(-krr_0\tau^2/R_1)}{2 \cos^2(\phi - \phi_0)/2 + i\tau^2} d\tau \quad (2.69)$$

This equation may be transformed into terms containing Fresnel integrals by using the relation (OTT 1943)

$$\int_{-\infty}^{\infty} \frac{e^{-kRs^2}}{s^2 - ia} ds = \frac{2\Gamma(n+\frac{1}{2})}{(kR)^n (\pm\sqrt{a})} \rho^n e^{-i\rho} \int_{\pm\sqrt{\rho}}^{\infty} \frac{e^{i\tau^2}}{\tau^{2n}} d\tau \quad (2.70)$$

where  $\rho = akR$

In our case we have to choose the minus-sign and thus

$$\underline{I}(-\phi_0) = \frac{e^{-i\pi/4}}{\sqrt{\pi}} \frac{e^{i(kR_1 - \sigma_-^2)}}{kR_1} \int_{\sigma_-}^{\infty} e^{i\tau^2} d\tau \quad (2.71)$$

where

$$\sigma_- = -\sqrt{\frac{2krr_0}{R_1}} \cos \frac{\phi - \phi_0}{2} \quad (2.72)$$

Eqs. (2.64), (2.71) and (2.43) now give us the total diffracted field in the form



$$\underline{p}^d = \frac{e^{-i\pi/4}}{\sqrt{\pi}} \frac{e^{ikR_1}}{kR_1} \{e^{-i\sigma_-^2} \underline{F}(\sigma_-) + e^{-i\sigma_+^2} \underline{F}(\sigma_+)\} \quad (2.73)$$

where the notation

$$\sigma_+ = -\sqrt{\frac{2krr_0}{R_1}} \cos \frac{\phi + \phi_0}{2} \quad (2.74)$$

has been introduced.

In most practical cases the conditions

$$\cos(\phi + \phi_0)/2 \gg \cos(\phi - \phi_0)/2 \quad (2.75)$$

$$\cos(\phi - \phi_0)/2 \ll 1 \quad (2.76)$$

are fulfilled and the second term in Eq. (2.73) can be neglected. The direct distance between source and receiver

$$R = \{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\}^{1/2} = \{R_1^2 - 4rr_0 \cos^2(\phi - \phi_0)/2\}^{1/2} \quad (2.77)$$

can then be approximated to

$$R \approx R_1 - \frac{2rr_0}{R_1} \cos^2(\phi - \phi_0)/2 \quad (2.78)$$

Thus under the conditions (2.75) and (2.76) the diffracted wave can be written

$$\underline{p}^d = \frac{e^{-i\pi/4}}{\sqrt{\pi}} \frac{e^{ikR}}{kR_1} \underline{F}(\sigma_-) \quad (2.79)$$

or in terms of the tabulated Fresnel functions

$$\underline{p}^d = \frac{e^{-i\pi/4}}{\sqrt{2}} \frac{e^{ikR}}{kR_1} \left\{ \frac{1}{2} - C\left(\sqrt{\frac{2}{\pi}} \sigma_- \right) + i \left( \frac{1}{2} - S\left(\sqrt{\frac{2}{\pi}} \sigma_- \right) \right) \right\} \quad (2.80)$$

In the following the minus-index on  $\sigma$  will be dropped when reference is made to either of the last two equations. The validity of the approximations





leading to Eq. (2.79) is investigated in Section 2.3.2.3.

### 2.3.2.2. The exact solution

The exact solution of our problem according to MACDONALD 1915 and SENIOR 1953 is

$$P^d = \frac{i}{2} \int_{-\mu_R}^{\infty} \frac{H_1^{(1)}}{R} (kR \cosh \mu) d\mu + \frac{i}{2} \int_{-\mu_S}^{\infty} \frac{H_1^{(1)}}{S} (kS \cosh \mu) d\mu$$

(2.81)

where the plus-sign is valid for a hard half-plane and the minus-sign for a soft one.  $H_1^{(1)}$  denotes first order Hankel functions of the first kind. With notations according to Fig. 2.8.

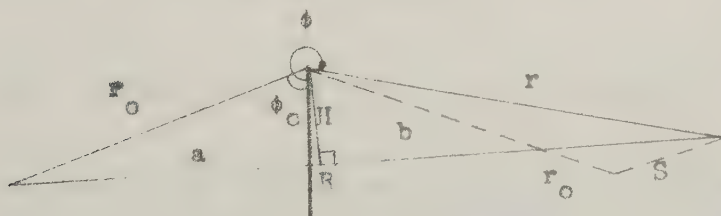


FIG. 2.8.  
Geometry of half-plane  
with incident and  
diffracted wave.

$$\mu_R = \sinh^{-1} \left( \frac{2\sqrt{rr_0}}{R} \cos \frac{1}{2} (\phi - \phi_0) \right) \quad (2.82)$$

$$\mu_S = \sinh^{-1} \left( \frac{2\sqrt{rr_0}}{S} \cos \frac{1}{2} (\phi + \phi_0) \right) \quad (2.83)$$

$$R^2 = r_0^2 + r^2 - 2rr_0 \cos(\phi - \phi_0) + (z - z_0)^2 \quad (2.84)$$

$$S^2 = r_0^2 + r^2 - 2rr_0 \cos(\phi + \phi_0) + (z - z_0)^2 \quad (2.85)$$

At about a wave length or more from the edge of the diffracting plane Eq. (2.81) may be approximated to

$$P^d = \frac{e^{-i\pi/4}}{\sqrt{\pi}} \left\{ \frac{e^{ikR}}{kR_x} F(x) + \frac{e^{ikS}}{kS_y} F(y) \right\} \quad (2.86)$$



where

$$R_x = \left( \frac{(R_1 + R)R_1}{2} \right)^{1/2} \quad (2.87)$$

$$x = -2 \left( \frac{krr_0}{R+R_1} \right)^{1/2} \cos(\phi - \phi_0)/2 = \{k(R_1 - R)\}^{1/2} \quad (2.88)$$

$$R_1^2 = (r + r_0)^2 + (z - z_0)^2 \quad (2.89)$$

$$S_y = \left( \frac{(R_1 + S)R_1}{2} \right)^{1/2} \quad (2.90)$$

$$y = -2 \left( \frac{krr_0}{S+R_1} \right)^{1/2} \cos(\phi + \phi_0)/2 \quad (2.91)$$

Unless we are very close to the half-plane  $y \gg 1$  and the second term in Eq. (2.86) may be neglected.

### 2.3.2.3. The validity of different solutions

The most common solutions are all of exactly the same form as Eq. (2.86) with the exception of the parameters  $x$ ,  $y$ ,  $R_x$  and  $S_y$ . For the sake of simplicity the second term in Eq. (2.86) will be neglected throughout and consequently only the discrepancies of  $x$  and  $R_x$  will be studied.

For the approximate solution in Section 2.3.2.1 the parameters are

$$R_x = R_1 \quad (2.92)$$

$$x = -2 \left( \frac{krr_0}{2R_1} \right)^{1/2} \cos(\phi - \phi_0)/2 = \left\{ k \frac{R_1 + R}{2R_1} (R_1 - R) \right\}^{1/2} \quad (2.93)$$

This solution will consequently be equivalent to the exact one as long as  $R_1 \approx R$ . As this must be the case whenever the second term in Eq. (2.86) can be neglected the accuracy of this approximate solution is satisfying.

REDFEARN 1940 derived a solution with the parameters

$$R_x = R \quad (2.94)$$



$$x = -2\left(\frac{krr_0}{2R}\right)^{1/2} \cos(\phi - \phi_0)/2 = \left\{k \frac{R_1 + R}{2R} (R_1 - R)\right\}^{1/2} \quad (2.95)$$

Hence it follows that this solution is equivalent to the exact one when  $R = R_1$ .

Kirchhoff's classical theory (See for example BAKER & COPSON 1950) is mostly used with the parameters written as

$$R_x = R \quad (2.96)$$

$$x = H\left\{\frac{k}{2}\left(\frac{1}{a} + \frac{1}{b}\right)\right\}^{1/2} \quad (2.97)$$

If  $H \ll a, b$  this solution is equivalent to Redfearn's.

The conclusion is that unless either the source or the receiver is situated very close to the half-plane the above solutions are equally useful.

#### 2.3.2.4. The nature of the diffracted wave

Neglecting the second term in Eq. (2.73) the diffracted field is given by

$$\underline{p}^d = \frac{e^{-i\pi/4}}{\sqrt{\pi}} \frac{e^{ikR_1}}{kR_1} e^{-i\sigma^2} \underline{F}(\sigma) \quad (2.98)$$

where

$$\sigma = -\left(\frac{2krr_0}{R_1}\right)^{1/2} \cos(\phi - \phi_0)/2 \quad (2.99)$$

according to Eq. (2.72).

Using the first term only in the asymptotic expansion at infinity of  $\underline{F}(\sigma)$  in Eq. (2.44)

$$\underline{F}_\infty(\sigma) = \frac{i}{2\sigma} e^{i\sigma^2} \quad (2.100)$$

Hence when  $\sigma \gg 1$





$$p_{\infty}^d = - \frac{e^{i\pi/4}}{2\sqrt{\pi}} \frac{e^{ikR_1}}{kR_1} \left( \frac{R_1}{2krr_0} \right)^{1/2} \frac{1}{\cos(\phi-\phi_0)/2} \quad (2.101)$$

As  $\cos(\phi-\phi_0)/2 < 0$  the phase angle

$$\psi_{\infty} = \pi/4 + kR_1 \quad (2.102)$$

This means that apart from a phase delay of  $\pi/4$  the phase is determined by the distance travelled over the edge of the screen.

When  $\phi-\phi_0$  is kept constant the amplitude will be distance dependent as

$$A_{\infty} = \frac{1}{\sqrt{rr_0 R_1}} \quad (2.103)$$

If we just consider the case when both source and receiver are situated on a straight line perpendicular to the half-plane ( $z = z_0$ )  $A_{\infty}$  simplifies to

$$A_{\infty} = \frac{1}{\sqrt{rr_0 (r+r_0)}} \quad (2.104)$$

What we are now primarily interested in is the behaviour of the wave on the receiver's side of the plane. By keeping  $r_0$  constant we see that if  $r \gg r_0$   $A_{\infty}$  behaves like a spherical wave radiating from the edge and if  $r \ll r_0$  it behaves like a cylindrical wave. In most practical cases concerning traffic noise  $A_{\infty}$  will behave as something in between. It must be observed that the words cylindrical and spherical have been used improperly because there is no rotational symmetry due to the factor  $\cos(\phi-\phi_0)/2$ .

An interesting aspect of Eq. (2.101) is that if we move parallel to the half-plane and  $(z-z_0) \gg (r+r_0)$  the diffracted field behaves like a proper cylindrical wave.

In the other asymptotic case when  $\sigma \ll 1$  the first two terms in the series expansion in Eq. (2.45) give

$$\underline{F}_0(\sigma) = \frac{\sqrt{\pi}}{2} e^{i\pi/4} - \sigma \quad (2.105)$$



and for the diffracted field

$$p_o^d = \frac{e^{-i\pi/4}}{\sqrt{\pi}} \frac{e^{i(kR_1 - \sigma^2)}}{kR_1} \left( \frac{\sqrt{\pi}}{2} e^{i\pi/4} - \sigma \right) \quad (2.106)$$

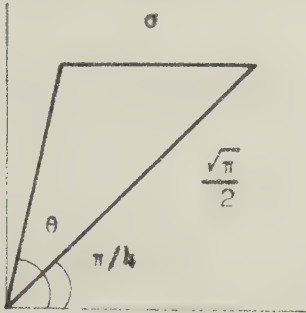


FIG. 2.9.  
Definition of the phase  
angle  $\theta$  in Eq. (2.107)

The phase angle

$$\psi_o = -\pi/4 + kR_1 - \sigma^2 + \theta \quad (2.107)$$

where  $\theta$  is found in Fig. 2.9. In the most extreme case when  $\sigma = 0$  Eq. (2.107) simplifies to

$$\psi_o = kR_1 \quad (2.108)$$

The distance dependence of the amplitude at a constant angle of diffraction is

$$A_o = \frac{1}{R_1} \quad (2.109)$$

that is like that of a spherical wave.

The nature of the diffracted wave can now be summarized in the equations

$$\frac{1}{\sqrt{rr_o R_1}} < A < \frac{1}{R_1} \quad (2.110)$$

$$kR_1 < \psi < kR_1 + \pi/4 \quad (2.111)$$





Eq. (2.110) indicates that the distance attenuation is larger than that of a cylindrical wave and less than or equal to that of a spherical wave where the equality is valid when  $\sigma \ll 1$  or  $r \gg r_0$ . Eq. (2.111) indicates that the phase is determined by the distance travelled source - edge of screen - receiver with an inaccuracy of  $\pi/4$  at most.

### 2.3.3. REFLECTED SPHERICAL WAVES

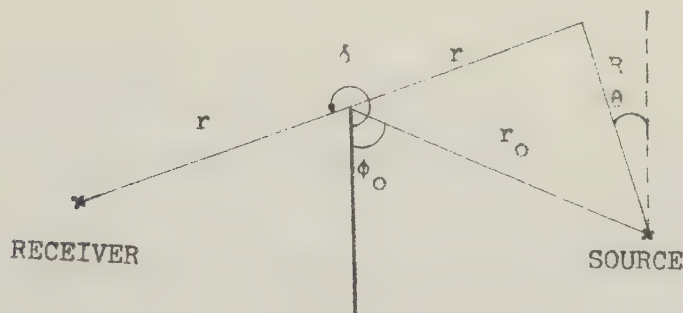


FIG. 2.10.  
Geometry of source and receiver for the reflected wave.

By repeating the analysis in Section 2.3.2.1 leading to Eq. (2.61) for the reflected wave it can be shown that the reflected diffracted wave takes the form

$$\underline{p}_R^d = \frac{1}{4\pi i} \int_{\pi/2}^{\pi} \underline{Q} \frac{e^{ikR}}{kR} \left\{ \frac{1}{\cos(\delta-\phi)/2} + \frac{1}{\cos(\delta+\phi)/2} \right\} d\delta \quad (2.112)$$

$\underline{Q}$  is obtained from Eqs. (2.11) and (2.14). Hence if  $t^2$  is neglected

$$\underline{Q} = kR \int_0^{\infty} e^{-kRt} \left( 1 - \frac{2u}{m+u} \left\{ 1 - \frac{kRt}{\rho} \right\}^{-1/2} \right) dt \quad (2.113)$$

where  $\underline{R}$  is given by Eq. (2.60) and

$$\underline{m} = \cos\theta \quad (2.114)$$

$$\underline{\rho} = ikR \frac{(\underline{m}+u)^2}{2(1+\underline{m}u)} \quad (2.115)$$

Inserting Eq. (2.113) into Eq. (2.112), distorting the  $\underline{\delta}$ -path to go through  $\underline{\delta} = \phi_0$  and then making the substitution  $\underline{\delta} = \underline{\delta}' + \phi_0$  yield



$$p_R^d = \underline{I}_R(-\phi_0) + \underline{I}_R(\phi_0) \quad (2.116)$$

where

$$\underline{I}_R(-\phi_0) = \frac{1}{4\pi i} \int_{P(0)} \int_0^\infty \left(1 - \frac{2u}{\underline{m}+u} \left\{1 - \frac{kRt}{\underline{\rho}}\right\}\right)^{-1/2} \frac{e^{ikR - kRt} d\underline{\delta}' dt}{\cos(\underline{\delta}' + \phi - \phi_0)/2} \quad (2.117)$$

$$\underline{m} = \frac{r \cos(\underline{\delta}' + \phi_0) + r_0 \cos \phi_0}{\{r^2 + r_0^2 + 2rr_0 \cos \underline{\delta}' + (z - z_0)^2\}^{1/2}} \quad (2.118)$$

The steepest descent substitution  $\cos \underline{\delta}' = 1 + i\tau^2$  and the usual approximation that  $\tau^2 \ll \tau$  now give

$$\begin{aligned} \underline{I}_R(-\phi_0) = & \frac{\sqrt{2} e^{-i\pi/4}}{4\pi i} \int_{-\infty}^{\infty} \int_0^\infty \left(1 - \frac{2u}{\underline{m}_0 + u} \left(1 - \frac{kRt}{\underline{\rho}_0}\right)\right)^{-1/2} \times \\ & \times \frac{e^{ikR} e^{-kRt} d\tau dt}{\cos(\phi - \phi_0)/2 - \frac{\tau}{2} \sqrt{-2i} \sin(\phi - \phi_0)/2} \end{aligned} \quad (2.119)$$

where

$$\underline{m}_0 = \frac{r+r_0}{R_1} \cos \phi_0 - \tau \frac{r}{R_1} \sqrt{-2i} \sin \phi_0 \quad (2.120)$$

$$\underline{R} = ((r+r_0)^2 + (z-z_0)^2 + 2irr_0 \tau^2)^{1/2} \approx R_1 + i \frac{rr_0}{R_1} \tau^2 \quad (2.121)$$

$$\underline{\rho}_0 = ikR \frac{(\underline{m}_0 + u)^2}{2(1 + \underline{m}_0 u)} \approx ikR_1 \frac{(\underline{m}_0 + u)^2}{2(1 + \underline{m}_0 u)} \quad (2.122)$$

Eq. (2.119) is not very suitable to use for computations. Without decreasing the accuracy significantly the  $t$ - and  $\tau$ -integrals may be separated by introducing the approximations which are given in Eqs. (2.121) and (2.122). Neglecting the second term in Eq. (2.116) on the same grounds as before the diffracted field can then be written in the form



$$p_R^d = \frac{e^{-i\pi/4}}{\sqrt{2\pi i}} \frac{e^{ikR_1}}{kR_1} \int_{-\infty}^{\infty} \underline{Q}_0(\tau) \frac{\exp(-krr_0\tau^2/R_1) d\tau}{2 \cos(\phi-\phi_0)/2 - \tau \sqrt{-2i} \sin(\phi-\phi_0)/2} \quad (2.123)$$

where

$$\underline{Q}_0(\tau) = \underline{R}_0 + (1 - \underline{R}_0) \underline{G}(\underline{\rho}_0) \quad (2.124)$$

$$\underline{R}_0 = \frac{\underline{m}_0 - \underline{v}}{\underline{m}_0 + \underline{v}} \quad (2.125)$$

and  $\underline{G}(\underline{\rho}_0)$  is given by Eqs. (2.21) and (2.22). Apart from the factor  $\underline{Q}_0(\tau)$  eq. (2.123) is of course equivalent to Eq. (2.69).

If the  $\tau$ -dependence of  $\underline{Q}_0$  could be eliminated in Eq. (2.123) then the ground effect could be separated from the diffraction effect. This will be possible when  $\underline{m}_0$  is independent of  $\tau$ , that is when

$$\sqrt{2}\tau \tan \phi_0 \ll 1 + \frac{r_0}{r} \quad (2.126)$$

The diffracted field can then be written as

$$p_R^d = \underline{Q}_0 \cdot p_{+1}^d \quad (2.127)$$

where  $p_{+1}^d$  = the diffracted field from a totally reflected image source and  $\underline{Q}_0 = \underline{Q}_0(0)$ . In principle this means spherical reflection at the angle  $\phi_0$  and the distance  $R_1$  between source and receiver.

It is difficult to find out the range of validity of Eq. (2.127) because  $\tau$  in Eq. (2.126) is a variable of integration. To get some idea about this problem computations have been carried out to compare Eqs. (2.123) and (2.127) in some different cases. The results which indicate that the validity of Eq. (2.127) is more far-reaching than might have been expected are given in Section 3.4.





## 2.4. The reflection of diffracted waves

This section treats the ground reflections on the receiver's side of the sound barrier.

### 2.4.1. NONREFLECTED WAVES

The wave-type to be considered is the one that was dealt with in Section 2.3.2.

According to the reciprocity theorem (See for example RAYLEIGH 1945, §109) the solution of this problem is given from Section 2.3.3 by letting source and receiver change places. Eq. (2.123) can then be used unmodified if Eq. (2.120) is exchanged for

$$\frac{m}{r} = \frac{r+r_o}{R_1} \cos \phi_r - \tau \frac{r_o}{R_1} \sqrt{-2i} \sin \phi_r \quad (2.128)$$

where

$$\phi_r = 2\pi - \phi \quad (2.129)$$

and the self-evident index changes are made.

Eqs. (2.126) and (2.127) transform into

$$\sqrt{2} \tau \tan \phi_r \ll 1 + \frac{r}{r_o} \quad (2.130)$$

$$p_R^d = Q_r p_{+1}^d \quad (2.131)$$

respectively.  $Q_r$  = the equivalent of  $Q_o$  but with  $\phi_o$  exchanged for  $\phi_r$ . In many cases  $r \gg r_o$  and the inequality can then be approximated to

$$\sqrt{2} \tau \frac{1}{H} \ll \frac{1}{r_o} \quad (2.132)$$

where  $H$  = the vertical distance between the top of the barrier and the image receiver. In this case Eq. (2.131) should be a very good approximation. This is not surprising. As was shown in Section 2.3.2.4 the diffracted wave should behave like a spherical wave when  $r \gg r_o$ .



## 2.4.2. REFLECTED WAVES

The wave-type to be considered is the one that was dealt with in Section 2.3.3.

Restricting to the partially separated Eq. (2.123) and its reciprocal equivalent the solution for the double reflected diffracted wave is given by

$$p_{RR}^d = \frac{e^{-i\pi/4}}{\sqrt{2\pi i}} \frac{e^{ikR_1}}{kR_1} \int_{-\infty}^{\infty} \underline{Q}_0(\tau) \underline{Q}_r(\tau) \frac{\exp(-krr_0\tau^2/R_1) d\tau}{2 \cos(\phi-\phi_0)/2 - \tau \sqrt{-2i} \sin(\phi-\phi_0)/2} \quad (2.133)$$

The totally separated solution becomes

$$p_{RR}^d = \underline{Q}_0 \underline{Q}_r p_{+1}^d \quad (2.134)$$

In some cases it may be convenient just to bring one of the functions  $\underline{Q}_0(\tau)$  and  $\underline{Q}_r(\tau)$  out of the integral sign in Eq. (2.133).

## 2.5. The reduction of sound by barriers on the ground

The total sound field in a point above the ground due to a point source situated on the other side of a barrier separating the two points is given by superposing the solutions which have been derived in Sections 2.3.2, 2.3.3, 2.4.1 and 2.4.2. As many of these solutions are unfit for computations it is desirable to make use of different approximations. This approximate procedure is also supported by the fact that there are many unpredictable variations in ground impedance and weather conditions that will affect the result and thus make even an "exact" solution less accurate. Accordingly the different types of solutions that are dealt with in this section have been approximated to the largest possible extent.



## 2.5.1. FINITE GROUND IMPEDANCE ON BOTH SIDES OF THE BARRIER

With indices according to Fig. 2.11 and letting the first index refer to

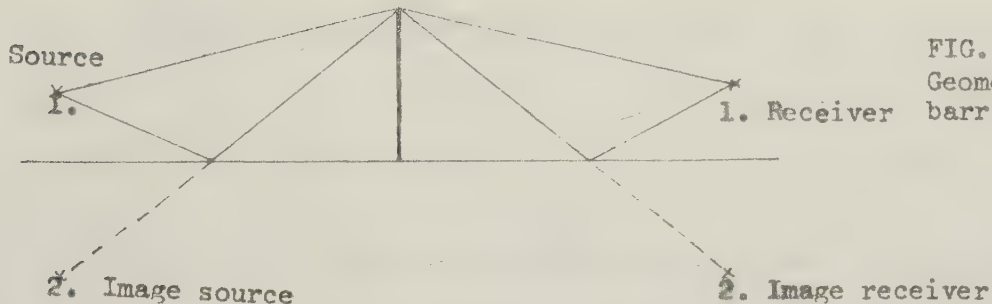


FIG. 2.11.  
Geometry of sound  
barrier on the ground.

the source side and the second index to the receiver side of the barrier superposition will give the total sound field  $p_t$  at the receiver. Thus

$$p_t = p_{11} + p_{12} + p_{21} + p_{22} \quad (2.135)$$

Neglecting the second diffraction term the most accurate solution is obtained by replacing  $p_{11}$ ,  $p_{21}$ ,  $p_{12}$  and  $p_{22}$  with Eqs. (2.79), (2.123), the reciprocal counterpart of (2.123) and (2.133) respectively. In the following this solution will be called the "exact" one although some minor approximations have been made.

Considering the separated solutions only  $p_{11}$ ,  $p_{12}$ ,  $p_{21}$  and  $p_{22}$  are given by Eqs. (2.79), (2.131), (2.127) and (2.134) respectively. Inserting these equations into Eq. (2.135) now yields

$$p_t = \sum_{i,j} \frac{e^{ikR_{ij}}}{k(R_1)_{11}} A_{ij} Q_{ij} \quad (2.136)$$

where

$$A_{ij} = \frac{(R_1)_{11}}{(R_1)_{ij}} \frac{e^{-i\pi/4}}{\sqrt{\pi}} F(\sigma_{ij}) \quad (2.137)$$

and from Eq. (2.93)

$$\sigma_{ij} = \left\{ k \frac{(R_1)_{ij} + R_{ij}}{2(R_1)_{ij}} ((R_1)_{ij} - R_{ij}) \right\}^{1/2} \approx \left\{ k((R_1)_{ij} - R_{ij}) \right\}^{1/2} \quad (2.138)$$





The other notations are self-explanatory.

As  $Q_{11} = 1$  and  $Q_{22} = Q_{12} \cdot Q_{21}$  Eq. (2.136) can be written as

$$p_t = \frac{e^{ikR_{11}}}{k(R_1)_{11}} (A_{11} + e^{ik(R_{21}-R_{11})} A_{21} Q_{21} + e^{ik(R_{12}-R_{11})} A_{12} Q_{12} + e^{ik(R_{22}-R_{11})} A_{22} Q_{12} Q_{21}) \quad (2.139)$$

The accuracy of this equation should be satisfying for most purposes.

Some comparisons between the "exact" solution and the approximate solution according to Eq. (2.139) are made in Section 3.5.2, where a great number of comparisons with measured values are also to be found.

#### 2.5.2. INFINITE GROUND IMPEDANCE ON ONE SIDE OF THE BARRIER

Inserting  $Q_{21} = 1$  into Eq. (2.139) gives

$$p_t = \frac{e^{ikR_{11}}}{k(R_1)_{11}} (A_{11} + A_{21} e^{ik(R_{21}-R_{11})} + A_{12} Q_{12} e^{ik(R_{12}-R_{11})} + A_{22} Q_{12} e^{ik(R_{22}-R_{11})}) \quad (2.140)$$

In many cases the sound source is situated close to the ground, that is  $R_{21} \approx R_{11}$ ,  $A_{11} \approx A_{21}$  and  $A_{12} \approx A_{22}$ . If we further assume that the ground level is the same on both sides,  $R_{22} = R_{11}$  and  $R_{12} = R_{21}$  and accordingly Eq. (2.140) is simplified to

$$p_t = 2 \frac{e^{ikR_{11}}}{k(R_1)_{11}} (A_{11} + Q_{12} A_{12}) \quad (2.141)$$

Calculated values from this equation are given in Section 3.6.2 for some different ground surfaces.

It should be observed that Eqs. (2.140) and (2.141) are very good approximations in the important practical case when there is an acoustic barrier close to a road ( $r_0$  not too large) and the receiver on the other side



is at a distance  $r \gg r_0$  from the barrier.

### 2.5.3. INFINITE GROUND IMPEDANCE ON BOTH SIDES OF THE BARRIER

One solution is immediately obtained from Eq. (2.140) by putting  $Q_{12} = 1$ . In this section a more descriptive type of solution will be derived.

Eq. (2.111) showed that the phase of the diffracted wave is determined by the factor  $kR_1$  with an inaccuracy of  $\pi/4$  at most. Putting, for the sake of simplicity, the source close to the ground and neglecting the above mentioned phase inaccuracy the total sound pressure becomes

$$p_t = 2 \frac{e^{ik(R_1)_{11}}}{k(R_1)_{11}} (|A_{11}| + |A_{12}| e^{ik\{(R_1)_{12} - (R_1)_{11}\}}) \quad (2.142)$$

if  $R_1 \approx R$  and common phase factors are cancelled.

In terms of the tabulated Fresnel functions C and S Eq. (2.142) can be written as

$$p_t = \sqrt{2} \frac{e^{ik(R_1)_{11}}}{k(R_1)_{11}} (M(\sqrt{\frac{2}{\pi}} \sigma_{11}) + M(\sqrt{\frac{2}{\pi}} \sigma_{12}) e^{ik\{(R_1)_{12} - (R_1)_{11}\}}) \quad (2.143)$$

where

$$M(x) = \left| \frac{1}{2} - C(x) + i\left(\frac{1}{2} - S(x)\right) \right| \quad (2.144)$$

The advantage of this approximate method is that it **is easy to realize**, what happens when the height of the receiver is varied. On the ground  $(R_1)_{12} = (R_1)_{11}$  and we will get a sound pressure maximum. When the height of the receiver is increased the factor  $k\{(R_1)_{12} - (R_1)_{11}\}$  will grow in magnitude and thus tend to decrease the sound pressure as long as the phase factor remains less than  $\pi$ .

Eqs. (2.142) and (2.143) are most suitable for low frequencies as the ground then often may be considered to be totally reflecting. Some calculations for 125 Hz are given in Section 3.6.3.



### 3. NUMERICAL CALCULATIONS AND MEASUREMENTS

The aim of this section is to test the validity of the presented theories and to give some illustrative examples that will facilitate the physical understanding of different sound propagation effects.

#### 3.1. Description of performed measurements

##### 3.1.1. APPARATUS

The sound source was a Goodman Axiom 201 12" loudspeaker as shown in Fig. 3.3 with directional characteristics according to Fig. 3.1.

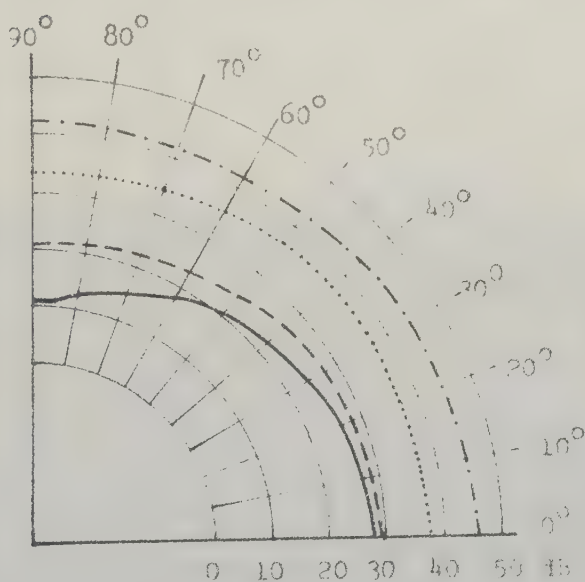


FIG. 3.1.  
Directional characteristics of  
the loudspeaker.

— 1000 Hz . . . 250 Hz  
- - 500 Hz - . - 125 Hz

The distance from the centre of the loudspeaker cone to the bottom of the cabinet was 0.27 m. The free field levels for the different measurement frequencies were measured at a distance of 2.00 m from the front of the loudspeaker cone in an anechoic chamber. The loudspeaker was driven by a constant sinus voltage, the frequency of which was checked by a frequency counter.

When a loudspeaker is situated close to a hard surface the power output is increased relative to the free field output. Theoretically (for details see, for instance MORSE-INGARD 1968) the maximum increase is 3 dB. For the type of sound source that the loudspeaker represents the performed measurements indicate, however, that this increase is of the order of







FIG. 3.2.  
Microphone lifting device  
in front of the cellular  
concrete wall.

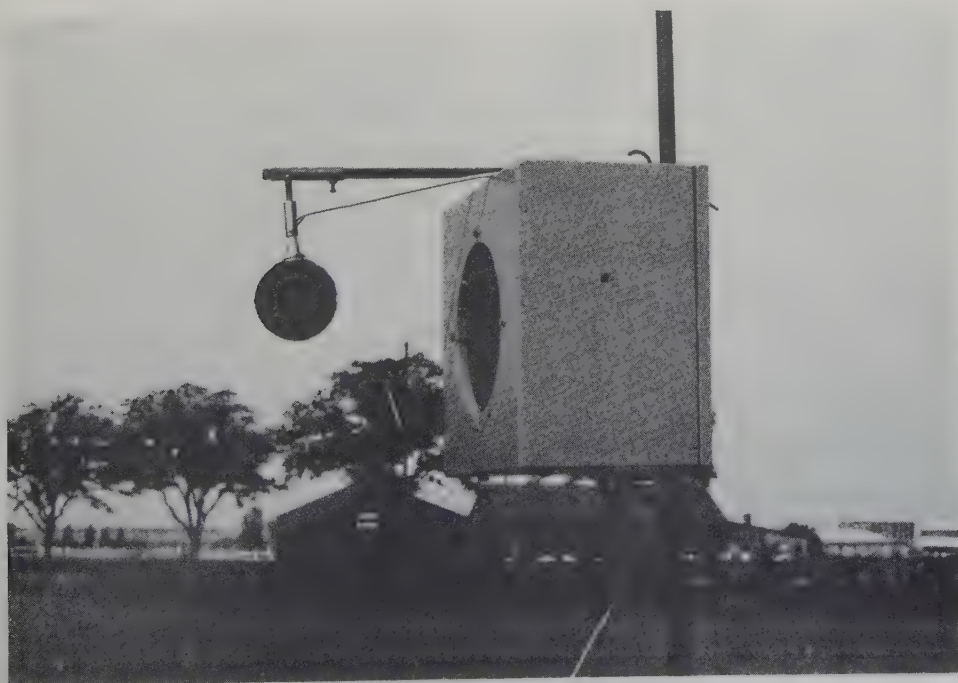


FIG. 3.3.  
Loudspeaker with microphone  
in calibration position.



magnitude 1-2 dB at 125 Hz and still less at higher frequencies. No corrections have been made for this effect.

The microphone was a 1" condenser microphone with a windscreen. In order to reduce the influence of the background sound level the signal was fed to the level recorder via a wave analyzer with a bandwidth of 6 Hz. The measurement chain was calibrated to give 100 dB at 1000 Hz and 15 V across the loudspeaker when the microphone was in a fixed calibration position as shown in Fig. 3.3. In this position the loudspeaker was 2 m above the ground level, that is all ground effects could be neglected.

By using a microphone lifting device as shown in Fig. 3.2 the sound level could be continuously recorded while the microphone was sweeping vertically. On every 10 cm of the lift stand a micro-switch on the microphone sledge touched a screw and thus put a mark on the recording paper. No correction has been made to take the size of the windscreen into consideration. This means that 5 cm should be added to all values of the receiver's height for the experimental curves. This is however insignificant.

### 3.1.2. MEASUREMENT SITES

All measurements with vertical microphone sweeps in the range 0-3.5 m were performed on measurement site 1 which is shown in Figs. 3.4 and 3.5. The area was covered by thinly grown grass and the soil was very hard and completely dry during all the measurements. The topography of the site appears from Fig. 3.6, where the position of the horizontal measurement range also is to be found.

The wall that was erected on measurement site 1 was 30 m wide, 2.8 m high and 0.15 m thick. The building material was cellular concrete.

The experiments with discrete microphone positions in the range 0-17 m above the ground level were made on measurement site 2 which is shown in Figs. 3.7 and 3.8. The ground surface was similar to the one of site 1 even if







FIG. 3.4. Measurement site 1 without wall from the south.



FIG. 3.5. Measurement site 1 with wall from the north.





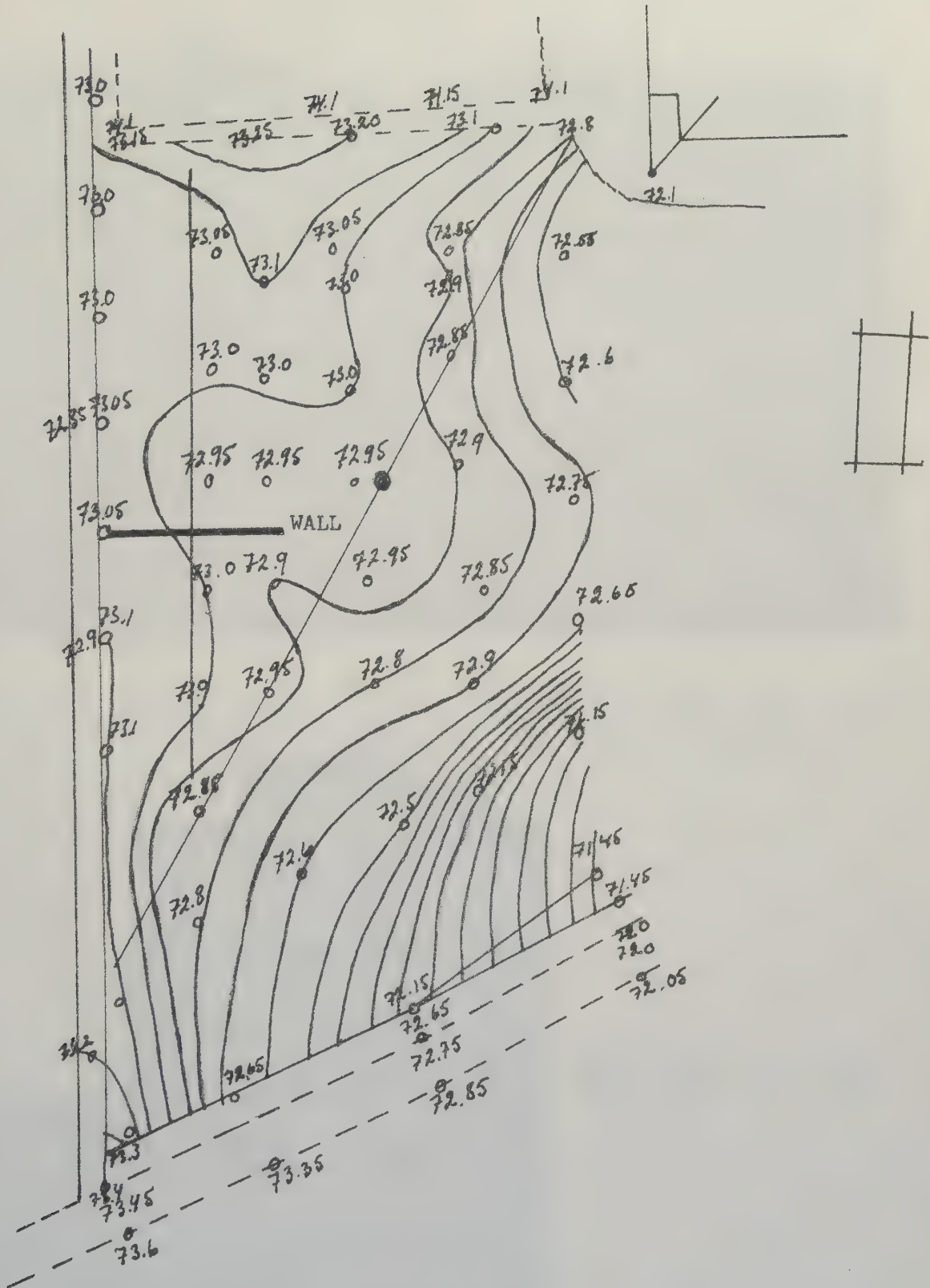


FIG. 3.6.

The topography of measurement site 1.

Scale: 1:1000





FIG. 3.7.  
Part of measurement site 2  
from the north.



FIG. 3.8.  
Measurement site 2 from  
the south.





the lawn of site 2 seemed to be in a slightly better condition. The vertical measurement range is indicated by an arrow on Fig. 3.6. The microphone was furnished with a windscreen and it was kept close to the brick-wall. Corrections have been made for the interference of the totally reflected sound wave from the vertical wall.

### 3.1.3. OTHER DETAILS ABOUT THE MEASUREMENTS

The measurements on site 1 were performed on two separate occasions, first in mid July without the wall and then in mid August with the wall. Each of the two measurement series was performed on 3 consecutive days. On both occasions the weather was sunny, the windspeed never exceeded 3 m/sec in any direction and the ground surface was extremely dry and hard. The grass was slightly longer during the barrier measurements. All measurements started very early in the morning and ceased late at night.

The microphone was calibrated a few times every day and two consecutive adjustments never exceeded 0.5 dB. The recorded dB-values have been read off for every 0.2 m and no decimals have been used.

Thermal wind turbulence caused considerable variations in sound pressure level at long distances. At 1000 Hz these variations could be of the order of magnitude  $\pm 3-4$  dB. As the sound pressure level was recorded continuously it has in most cases been possible to smooth out these variations.

The measurement points were exactly the same both with and without the wall.

On measurement site 2 the sound pressure level was recorded for about 10 seconds at each microphone height and the recorded curve was then visually averaged.

The source height (SH) of the loudspeaker has been defined as the distance from the centre of the loudspeaker cone to the ground level.

### 3.2. The propagation of sound over level grassland

The reference sound pressure level is always the free field level + 6 dB, that is the level that would be obtained close to a surface with infinite





impedance. The horizontal distance between source and receiver and the height above the ground level of source and receiver are denoted  $D$ ,  $SH$  and  $RH$  respectively.

### 3.2.1. COMPARISONS BETWEEN CALCULATED AND MEASURED VALUES FOR SPHERICAL WAVES

For the computations Eqs. (2.16), (2.17) and (2.20)-(2.24) have been used. The different values of the admittance  $\underline{u}$  were obtained by applying the trial and error method that was outlined in Section 2.1.2. For each frequency the computations have been carried out for two different admittances, one of which matched the experimental curves from measurement site 1 best while the other matched those of site 2. As an indirect method has been used to get the values of  $\underline{u}$  a great number of examples have been included.

The experimental values of Figs. 3.9-3.56 come from measurement site 1 while those of Figs. 3.57-3.68 come from site 2. The theoretical values have been computed in intervals of 0.2 m and 1.0 m for site 1 and 2 respectively. These intervals are equivalent to those which were used when evaluating the experimental recordings.

Even if the curves in the figures that follow speak for themselves a few comments may be worthwhile.

The paradoxical experimental value of +1 dB at 125 Hz for low source heights has been explained in Section 3.1.1.

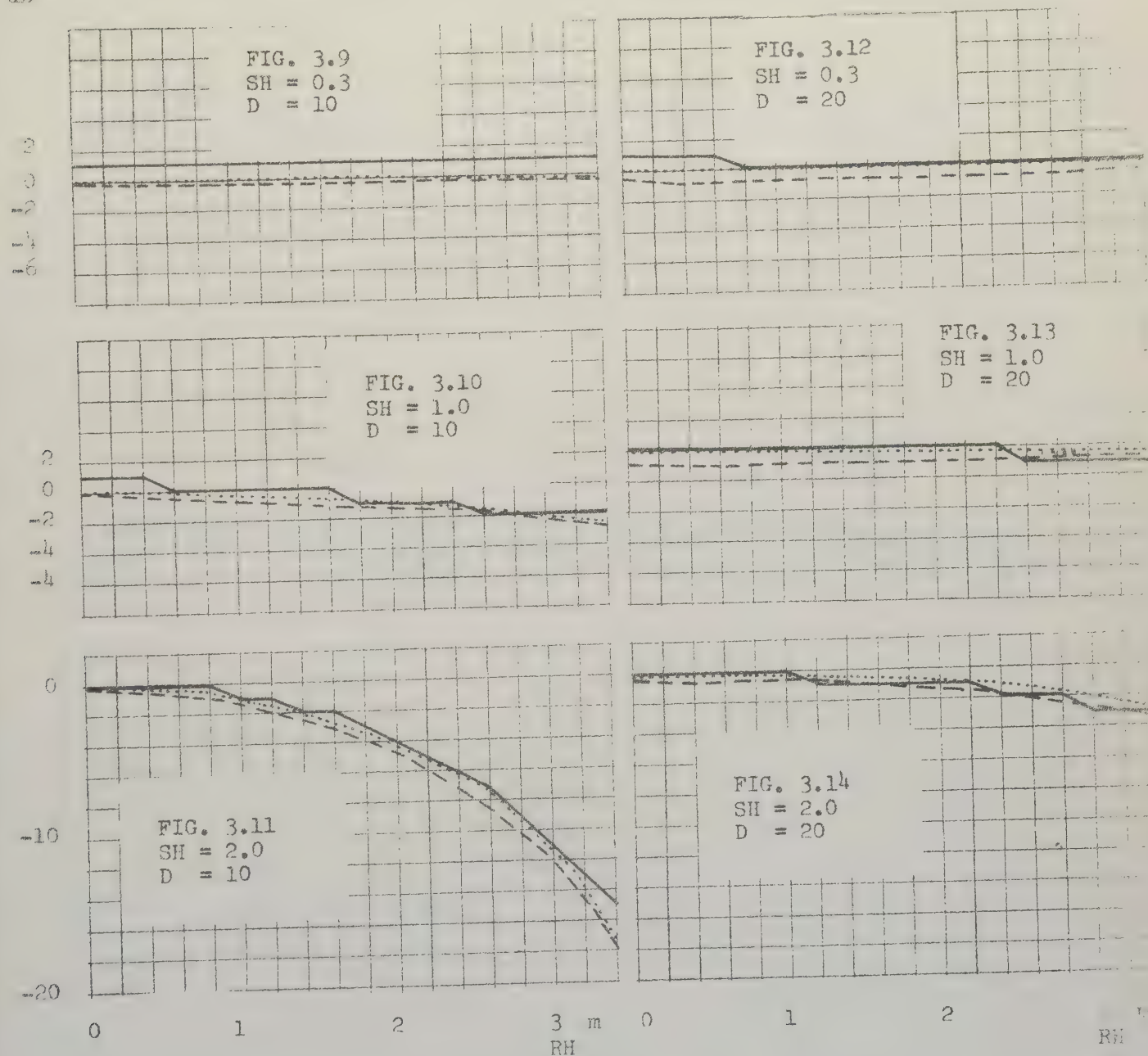
As is illustrated by, for example, Figs. 3.23 and 3.32 a small change in ground admittance is most critical at long distances and low receiver heights. Since a very large ground area is "actively" reflecting the incident sound waves in these cases and local variations in ground admittance are likely to occur within this area it might be difficult to obtain good agreement between theory and experiments. This might be an explanation to the less satisfying agreement in, for example, Figs. 3.60-3.64 for low receiver heights. This explanation is also supported by the fact that the discrepancies are largest for low frequencies, that is when the reflecting area is largest.



It should be observed that the ground admittance is extremely small at 125 Hz. This means that we may consider the ground surface to be totally reflecting and still retain a reasonable accuracy in the computations. This will at least be the case at moderate distances.

As a rule the agreement between measured and calculated values is satisfactory.

dB



FIGS. 3.9 - 3.14. 125 Hz

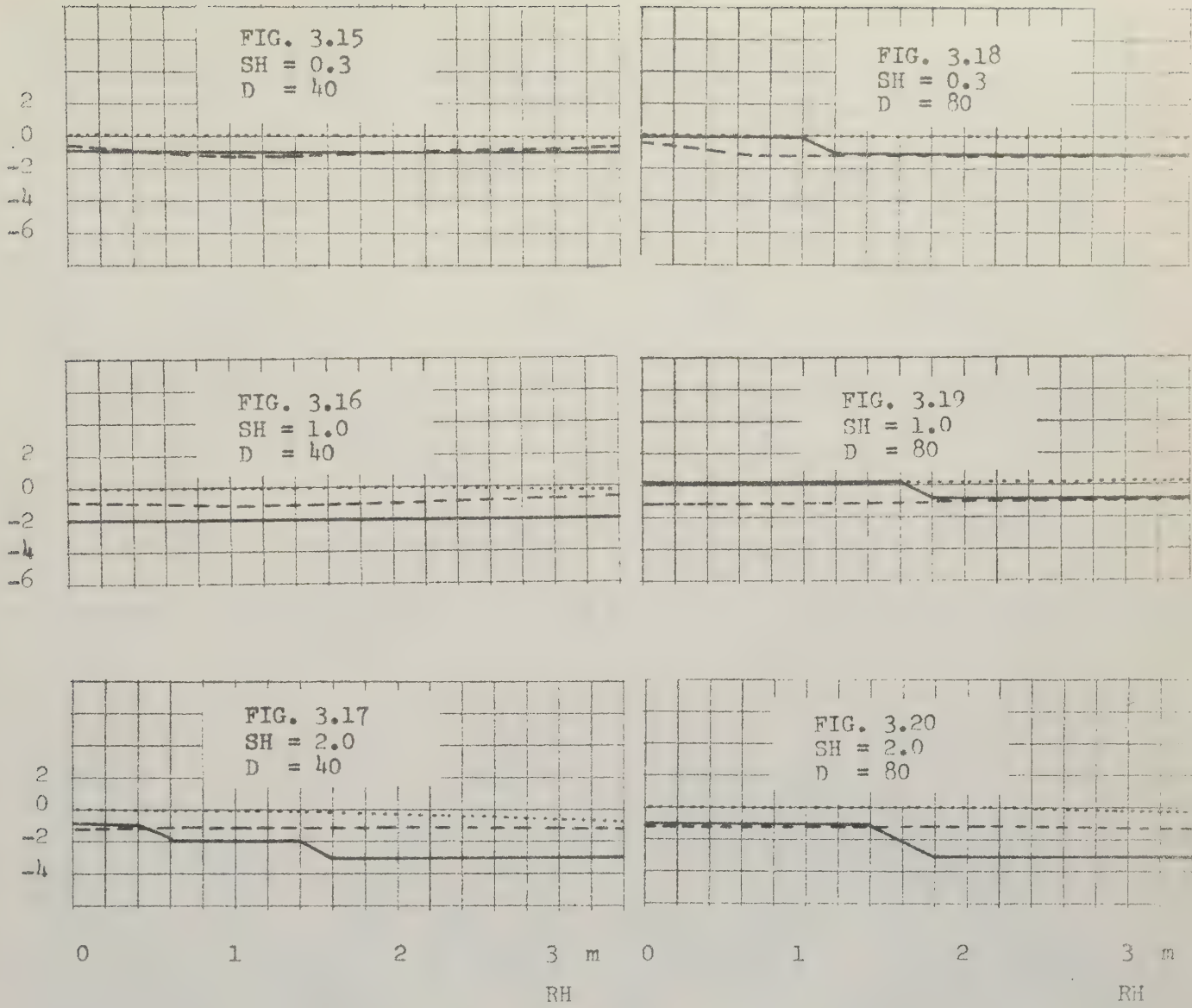
— measured

- - -  $\underline{v} = 0.005 - i \cdot 0.005$

. . .  $\underline{v} = 0.000 - i \cdot 0.000$



dB



FIGS. 3.15-3.20

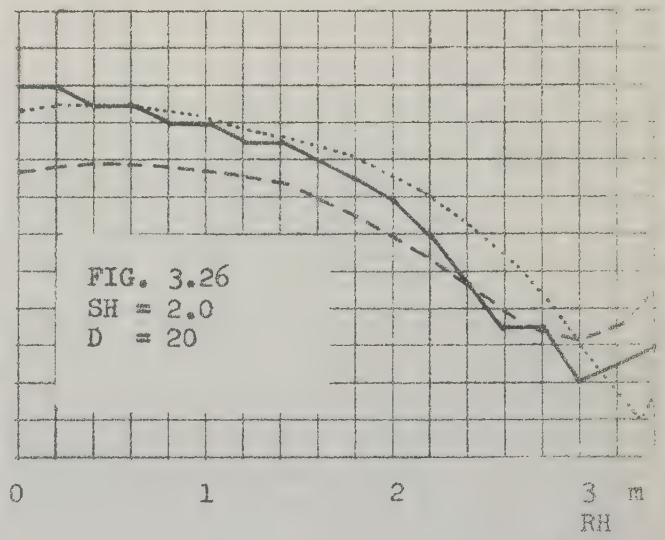
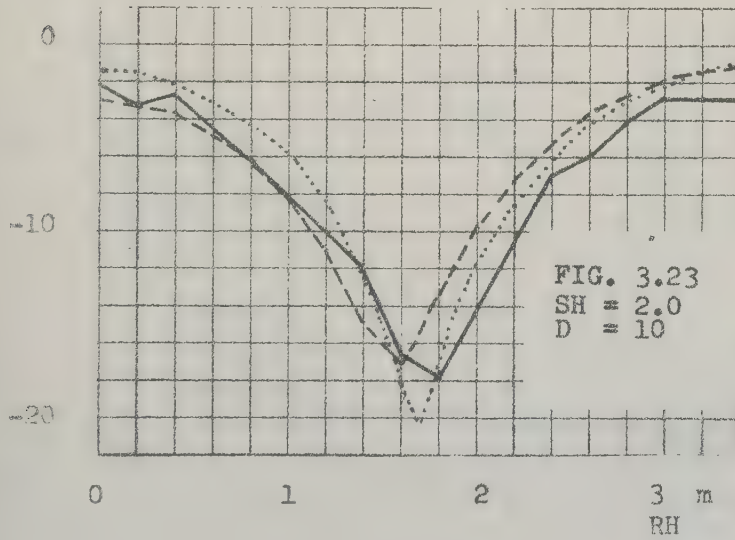
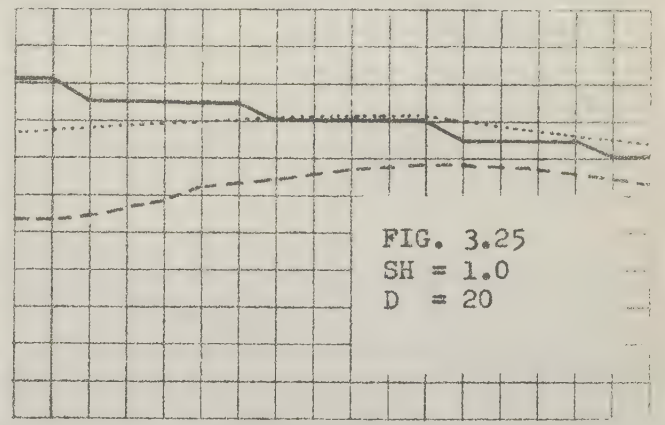
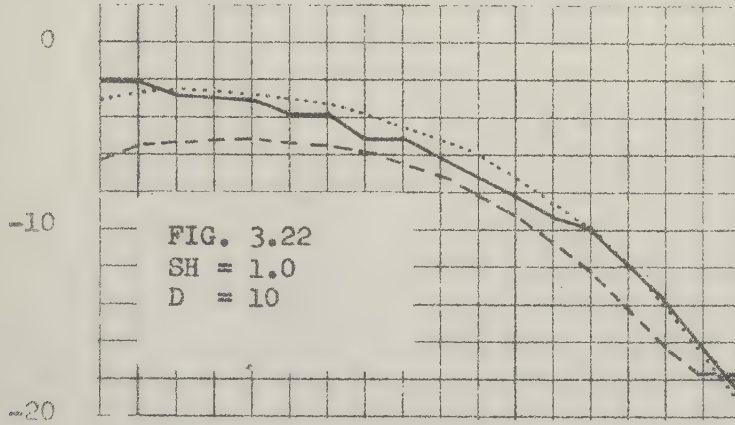
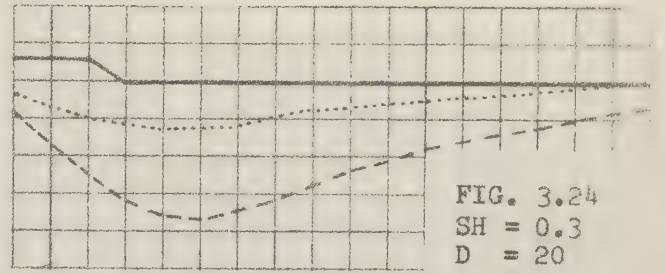
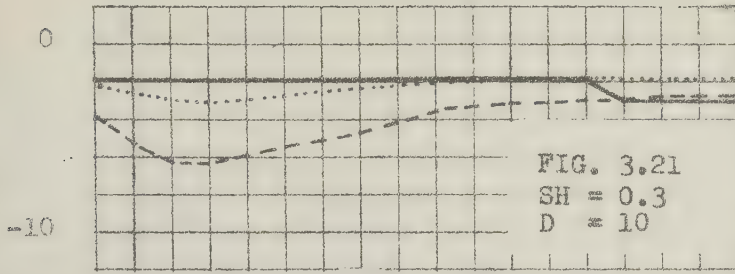
125 Hz

— measured  
 - - -  $\underline{v} = 0.005 - i \cdot 0.005$   
 . . .  $\underline{v} = 0.000 - i \cdot 0.000$





dB



FIGS. 3.21 - 3.26.

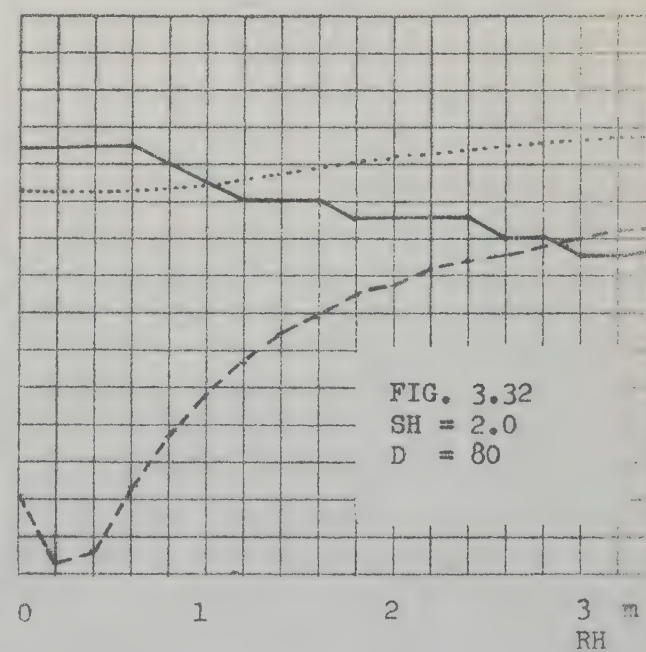
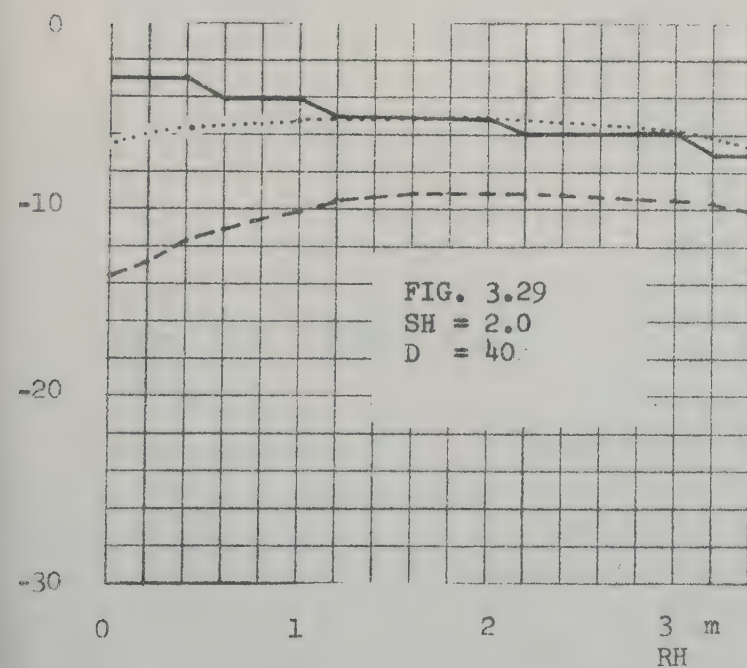
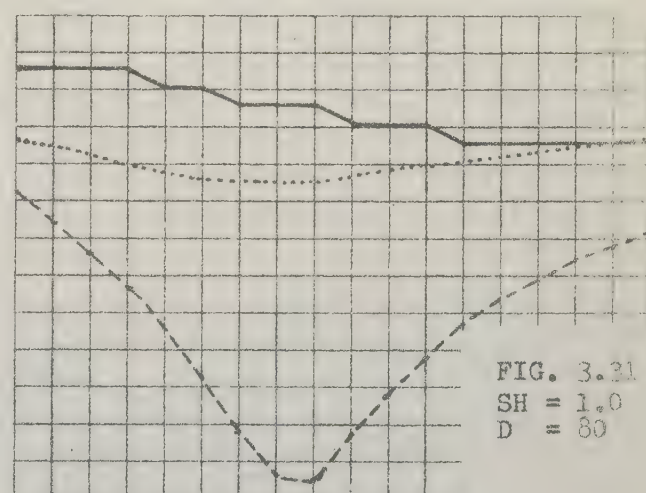
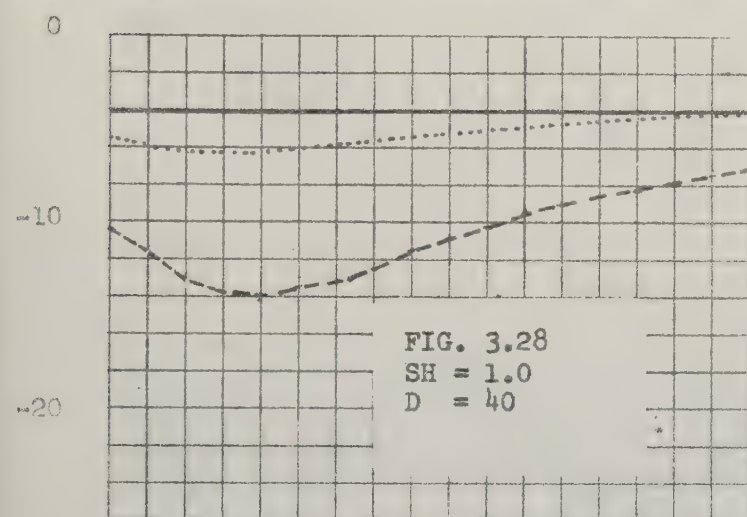
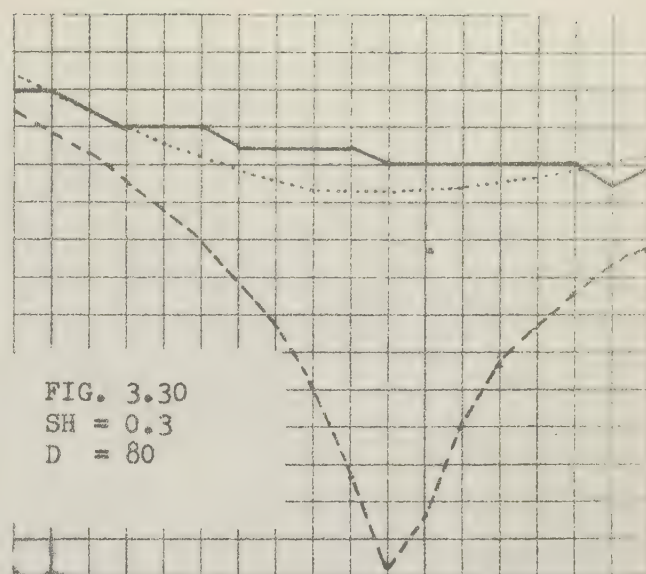
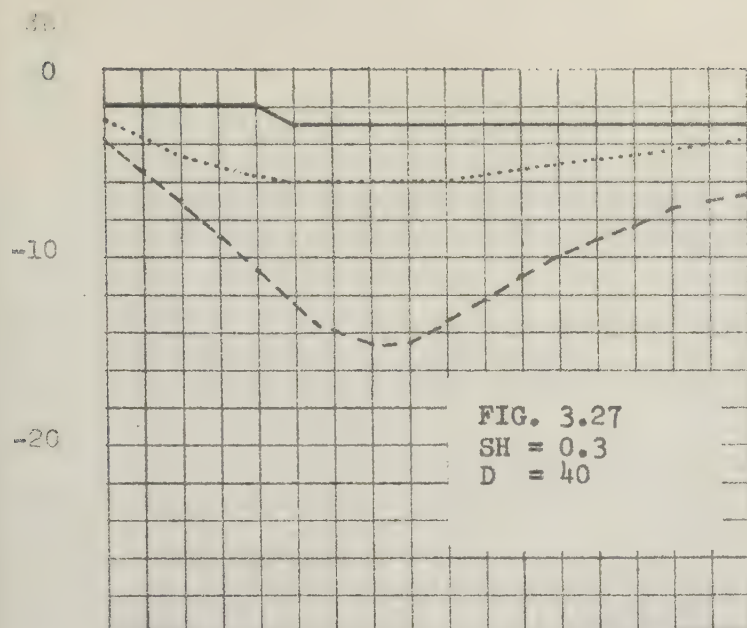
250 Hz

— measured

- - -  $\underline{u} = 0.05 - i \cdot 0.05$

. . .  $\underline{u} = 0.03 - i \cdot 0.02$



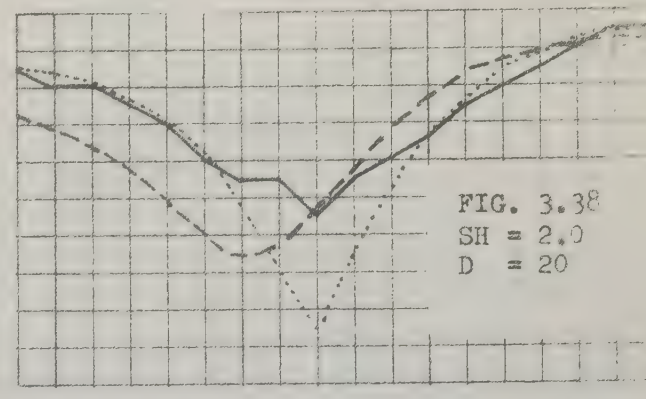
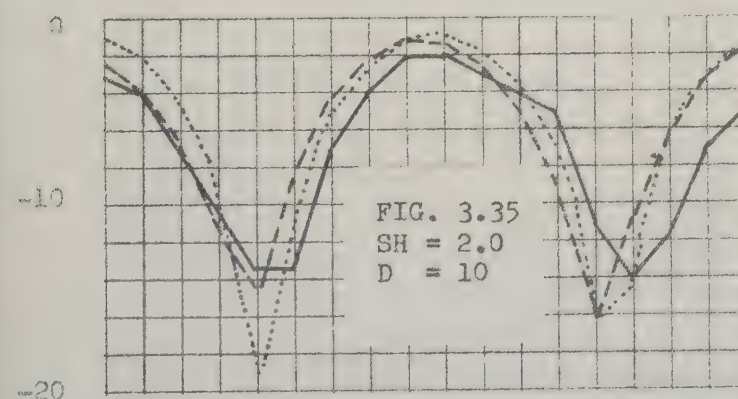
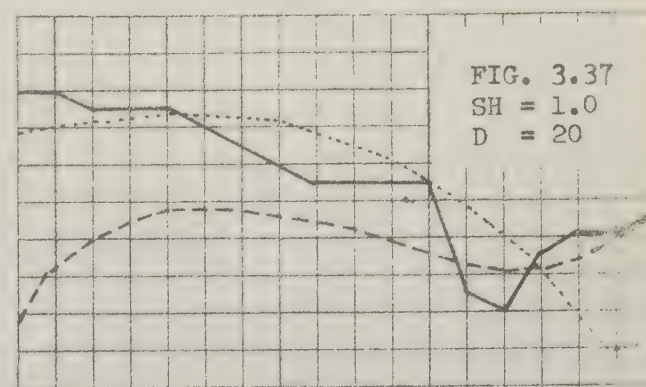
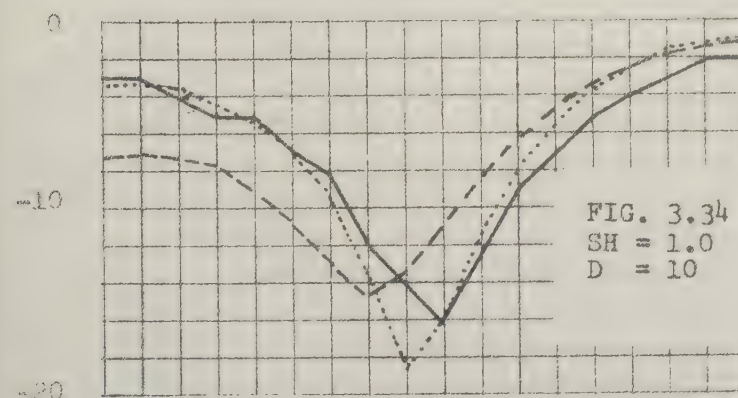
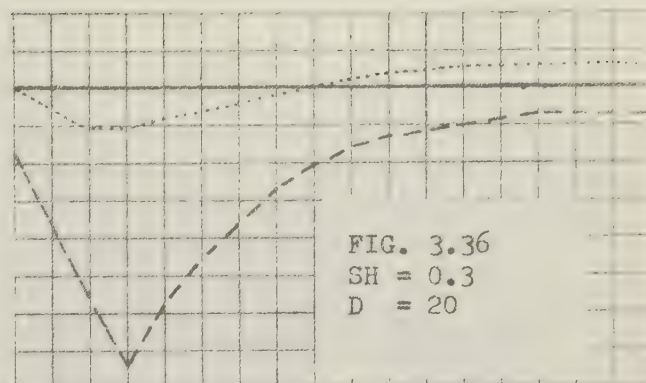
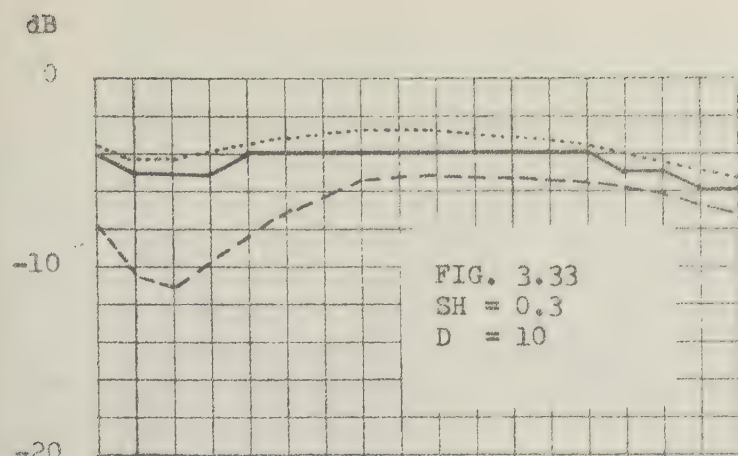


FIGS. 3.27 - 3.32. 250 Hz

— measured    - - -  $\underline{u} = 0.05 - i \cdot 0.05$     . . .  $\underline{u} = 0.03 - i \cdot 0.02$







0 1 2 3 m RH

0 1 2 3 m RH

FIGS. 3.33 - 3.38.

500 Hz

— measured

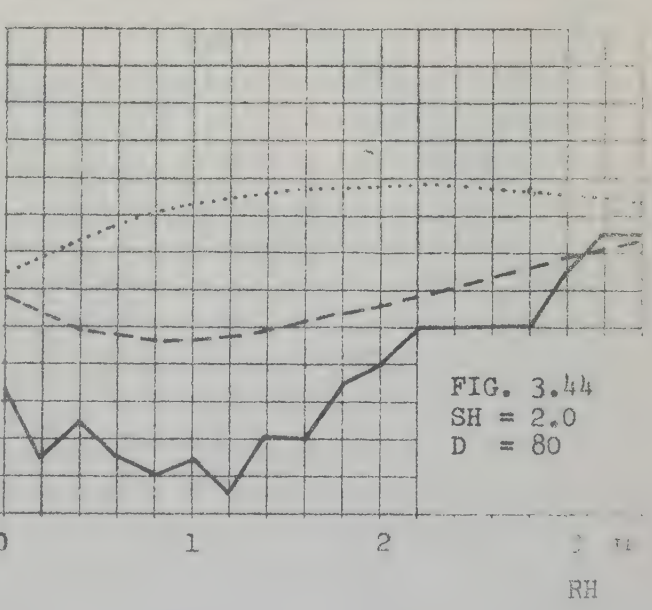
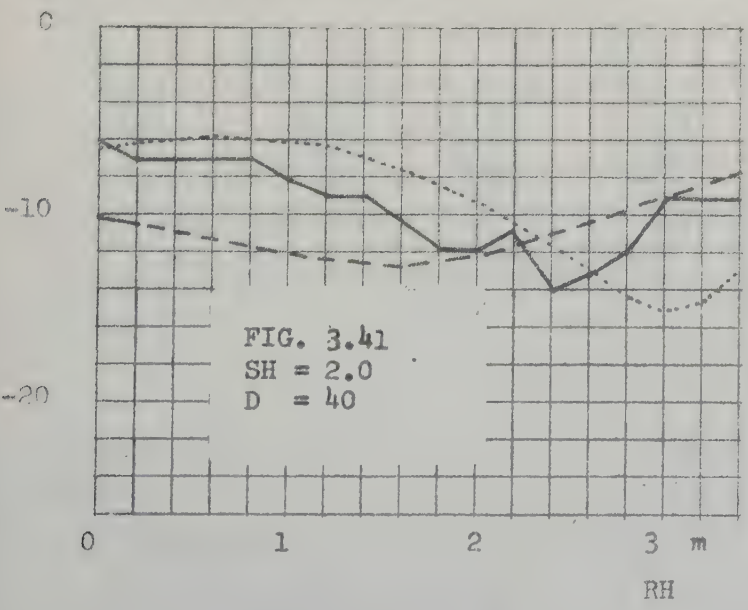
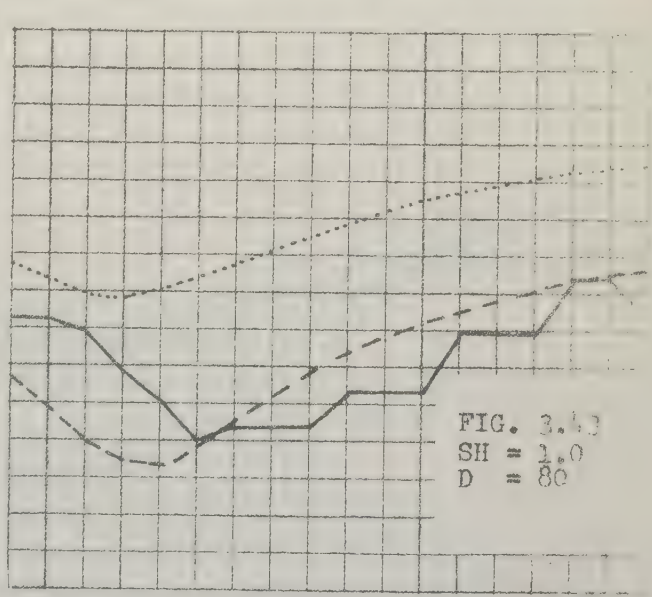
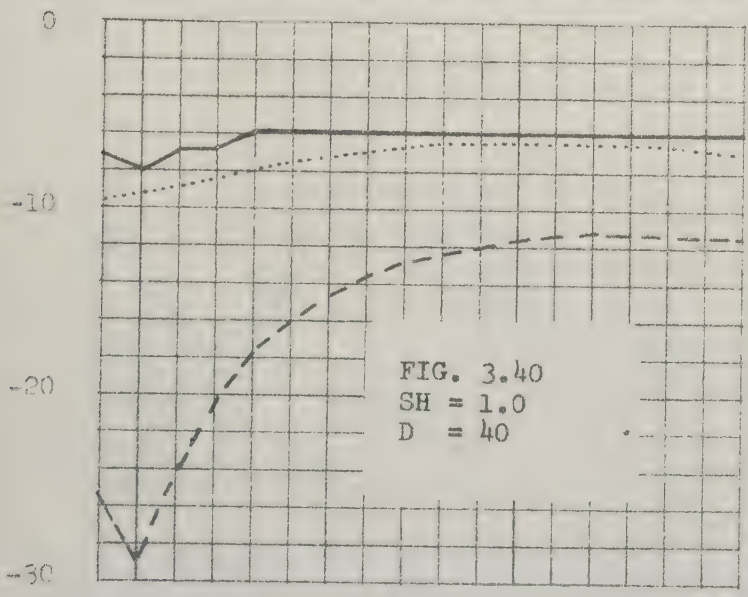
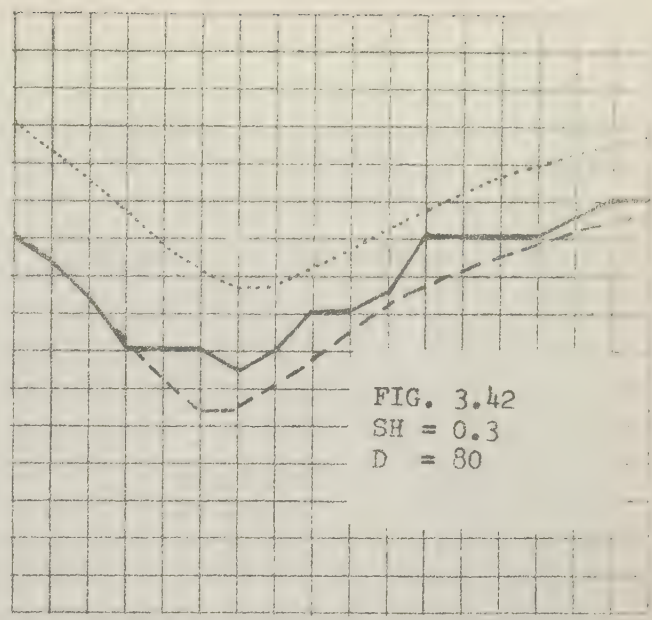
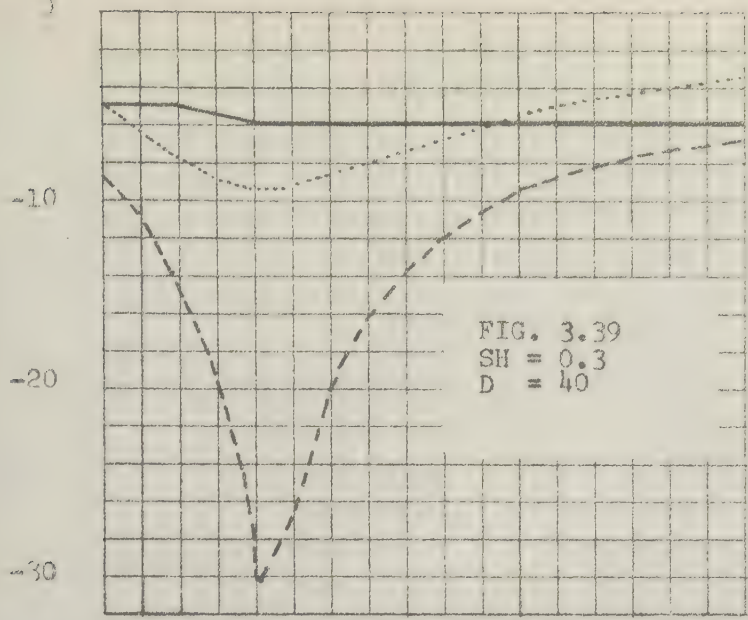
---  $\underline{u} = 0.06 - i \cdot 0.06$

...  $\underline{u} = 0.03 - i \cdot 0.02$





dB



FIGS. 3.39 - 3.44. 500 Hz

— measured    - - -  $\underline{u} = 0.06 - i \cdot 0.06$     . . .  $\underline{u} = 0.03 - i \cdot 0.02$



10

0

-10

-20

-30

FIG. 3.45  
SH = 0.3  
D = 10

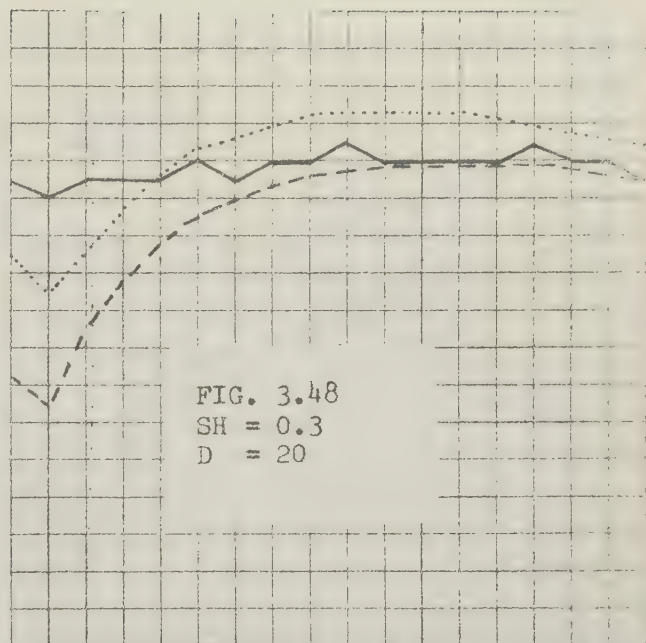


FIG. 3.48  
SH = 0.3  
D = 20

0

-10

FIG. 3.46  
SH = 1.0  
D = 10

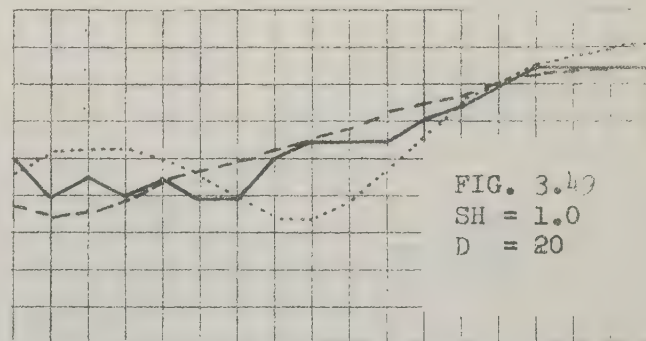


FIG. 3.49  
SH = 1.0  
D = 20

0

-10

-20

FIG. 3.47  
SH = 2.0  
D = 10

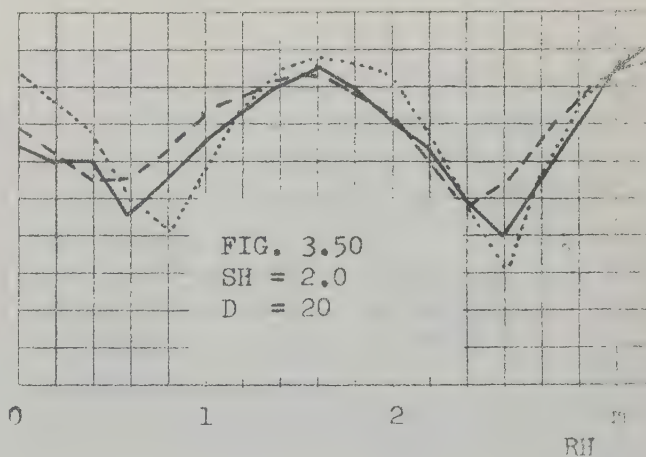


FIG. 3.50  
SH = 2.0  
D = 20

0 1 2 3 m RH

0 1 2 3 m RH

FIGS. 3.45 - 3.50.

1000 Hz

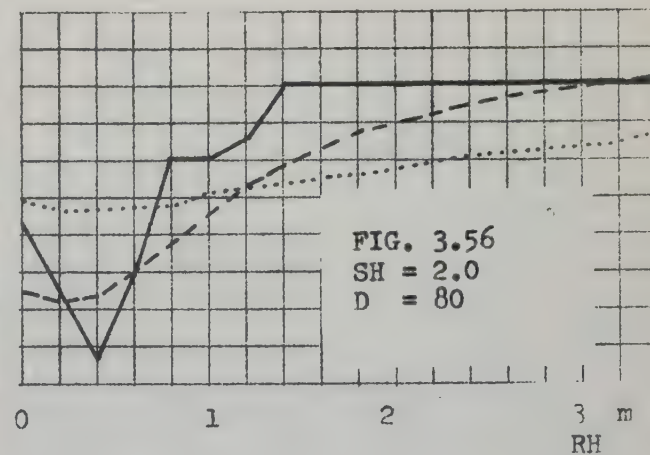
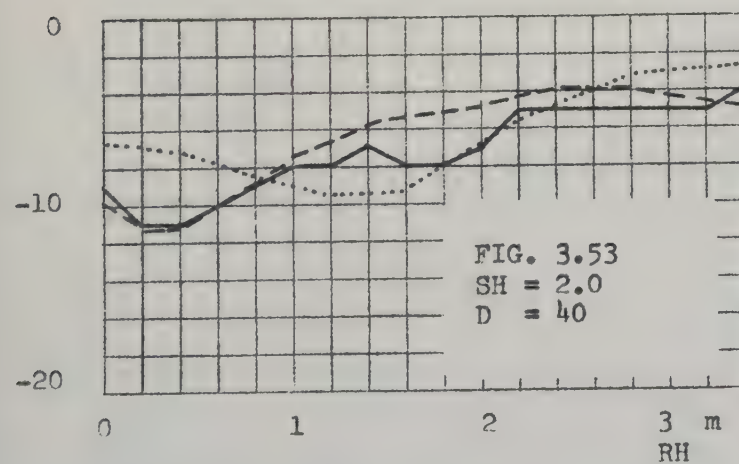
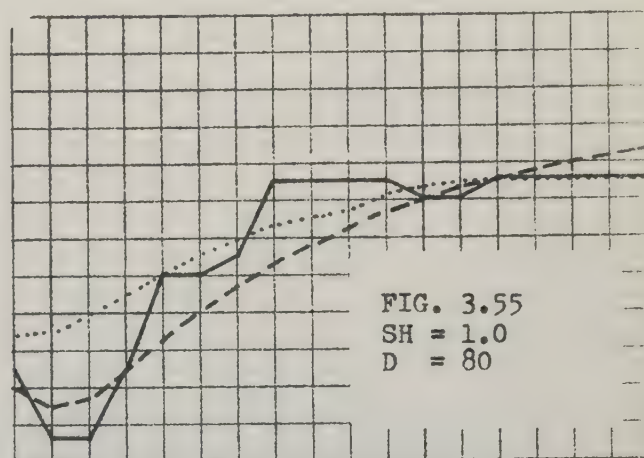
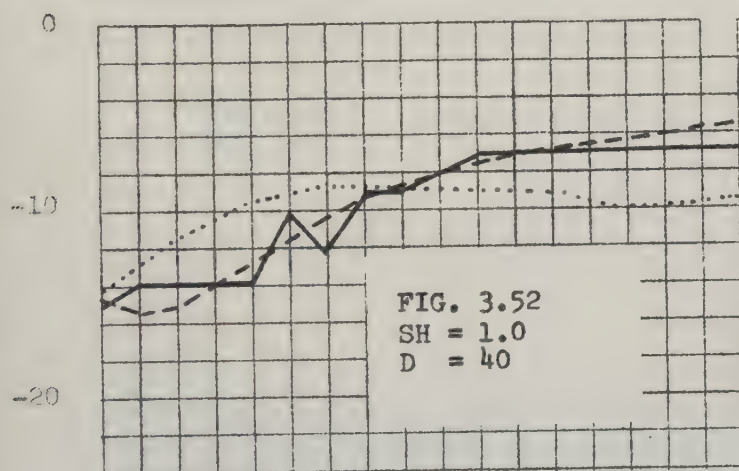
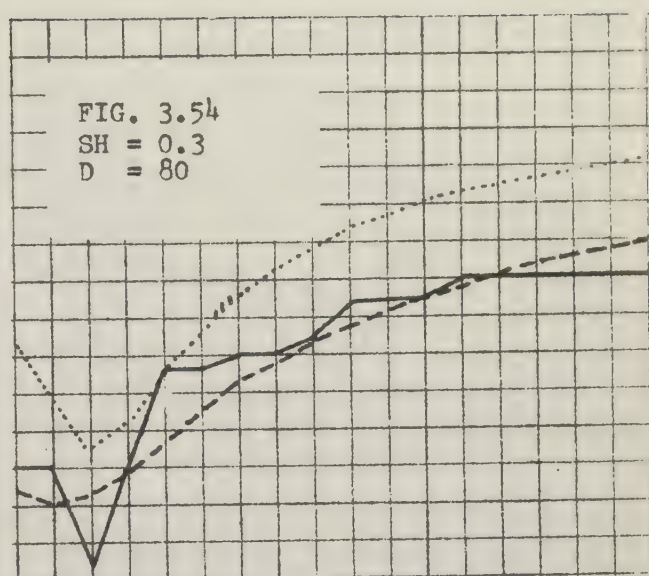
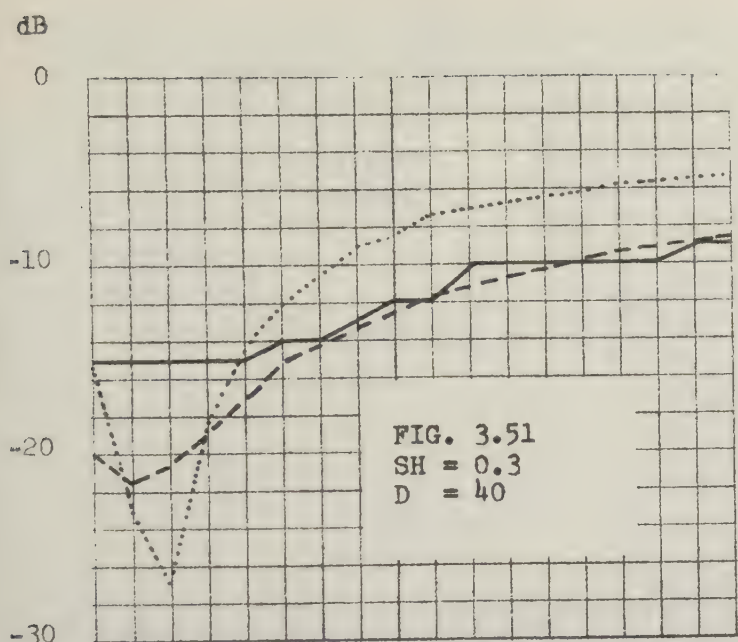
— measured

- - -  $\underline{u} = 0.10 - i \cdot 0.05$

. . .  $\underline{u} = 0.05 - i \cdot 0.02$







FIGS. 3.51 - 3.56, 1000 Hz

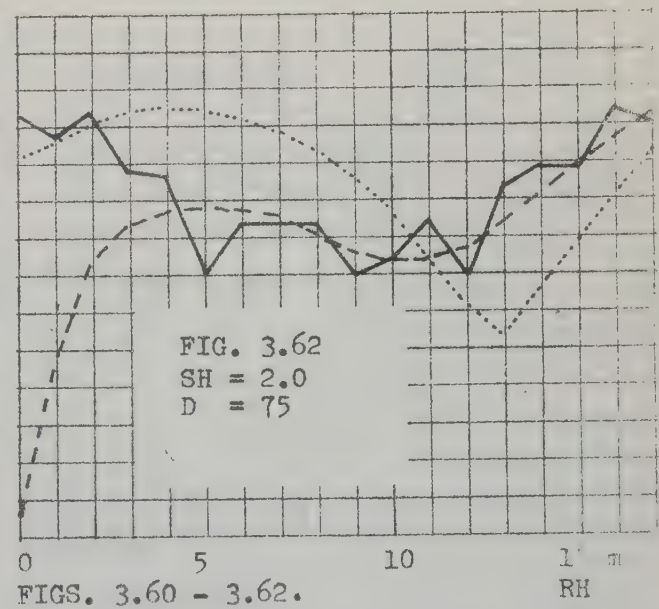
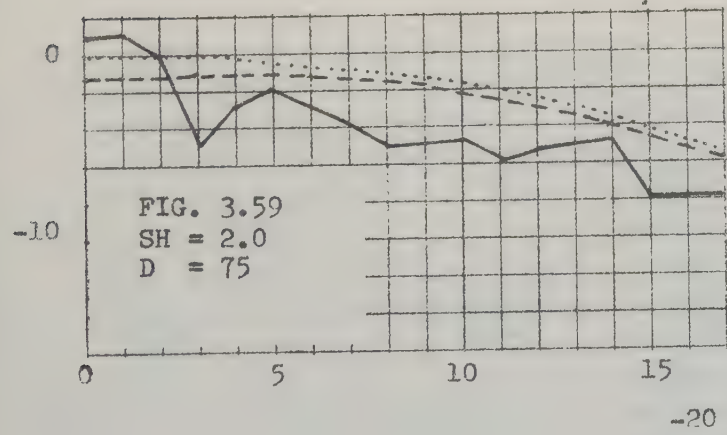
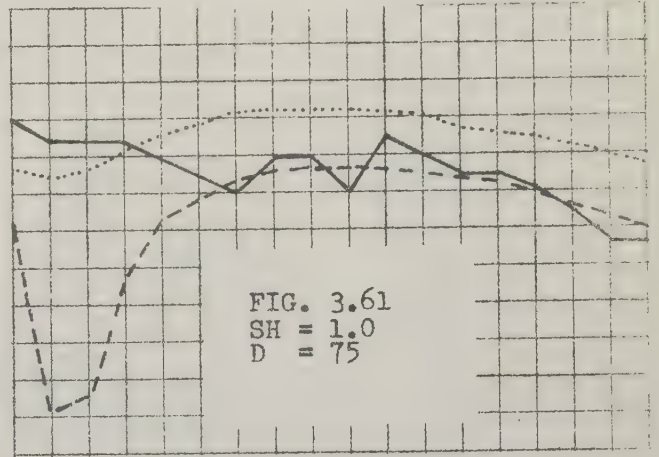
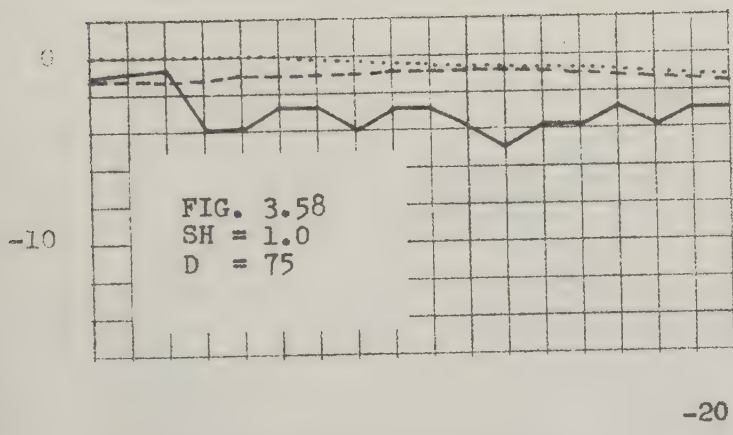
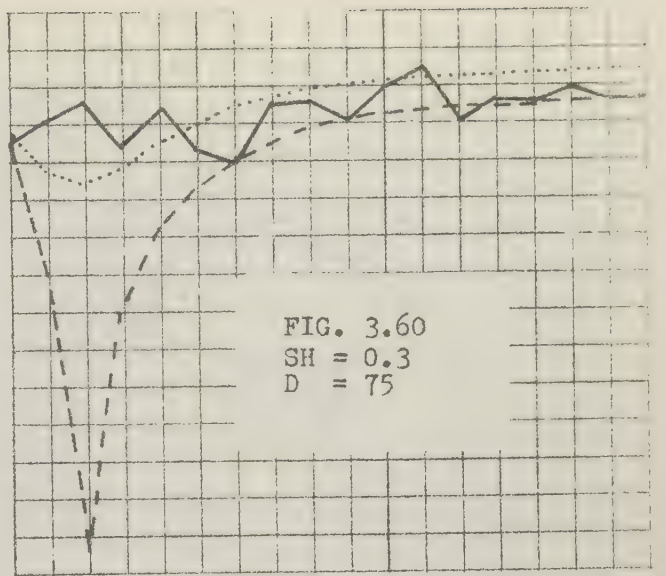
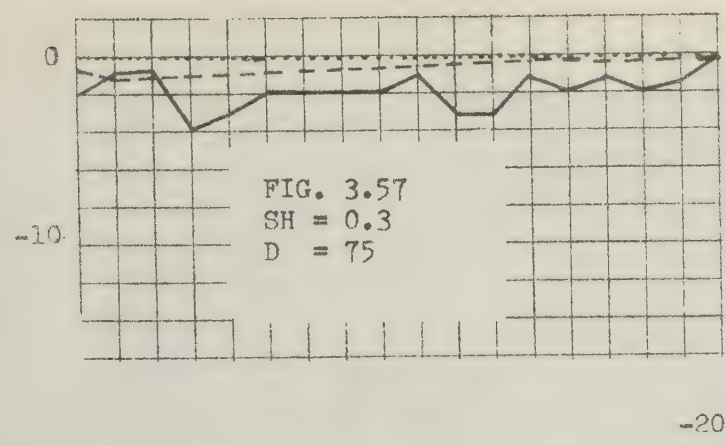
— measured

---  $\bar{u} = 0.10 - i \cdot 0.05$ ...  $\bar{u} = 0.05 - i \cdot 0.02$





dB



FIGS. 3.57 - 3.59.

125 Hz

— measured  
---  $\underline{u} = 0.005 - i \cdot 0.005$   
...  $\underline{u} = 0.000 - i \cdot 0.000$

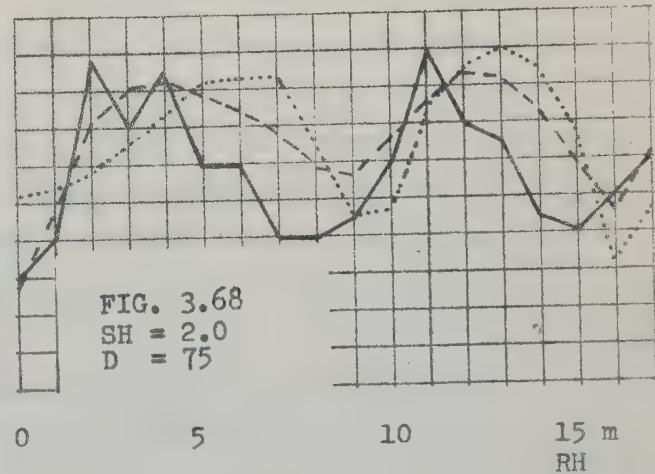
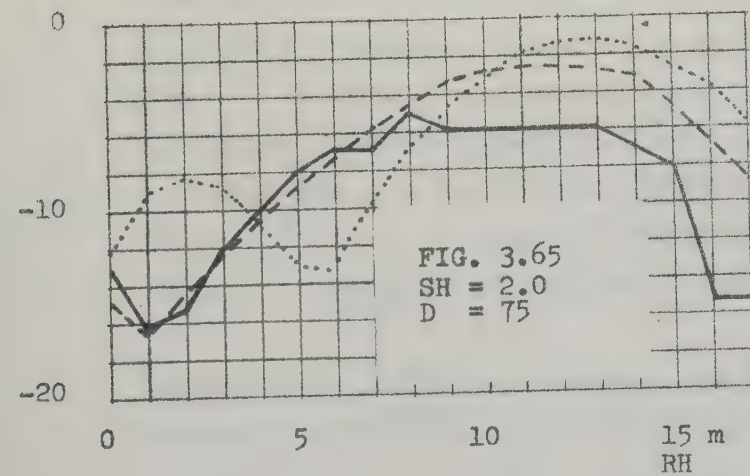
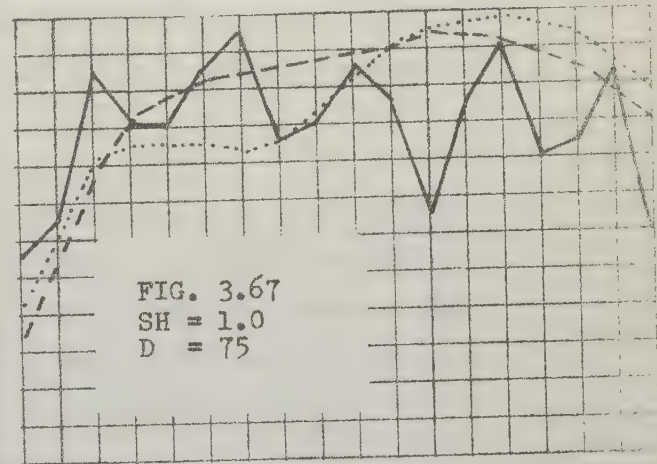
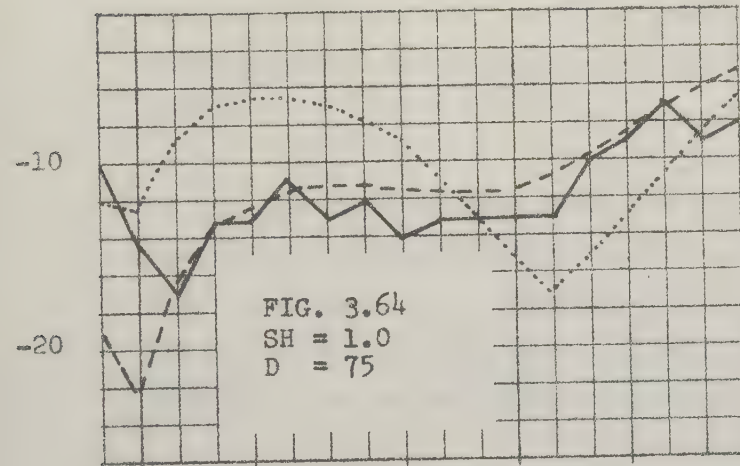
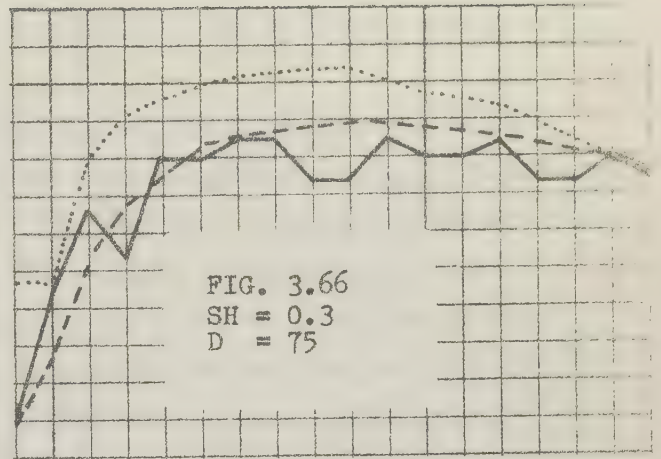
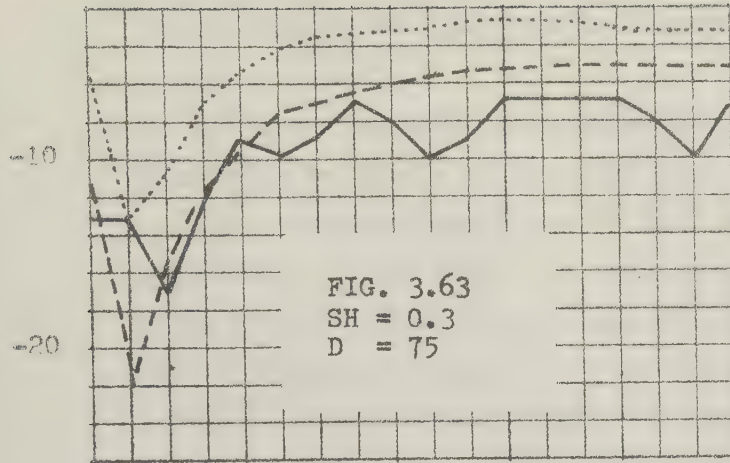
FIGS. 3.60 - 3.62.

250 Hz

— measured  
---  $\underline{u} = 0.05 - i \cdot 0.05$   
...  $\underline{u} = 0.03 - i \cdot 0.02$



dB



FIGS. 3.63 - 3.65.  
500 Hz

— measured  
- - -  $\underline{u} = 0.06 - i \cdot 0.06$   
...  $\underline{u} = 0.03 - i \cdot 0.02$

FIGS. 3.66 - 3.68.  
1000 Hz

— measured  
- - -  $\underline{u} = 0.10 - i \cdot 0.05$   
...  $\underline{u} = 0.05 - i \cdot 0.02$





### 3.2.2. CALCULATED COMPARISONS BETWEEN CYLINDRICAL AND SPHERICAL WAVES

It was shown in Section 2.3.2.4 that the diffracted wave in some cases behaves more like a cylindrical wave than like a spherical one. In Section 2.4 the separated solutions (2.131) and (2.134) both contain spherical reflection coefficients. By combining the results of Section 2.3.2.4 with a study of the propagation of cylindrical waves in relation to the one of spherical waves it is possible to get an idea of the validity of these solutions. Thus it may be of some interest to study cylindrical waves even if these hardly appear in real life. In this section it will only be shown in some specific examples that there may be significant differences in the propagation properties of cylindrical and spherical waves.

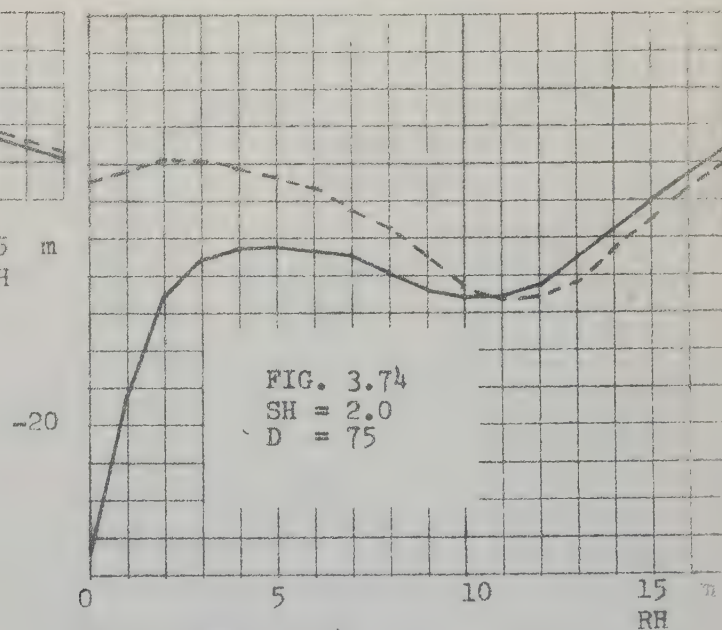
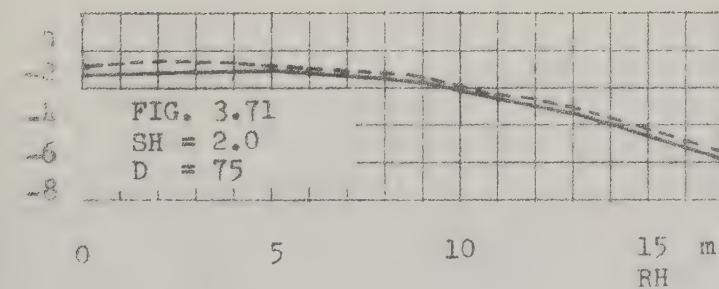
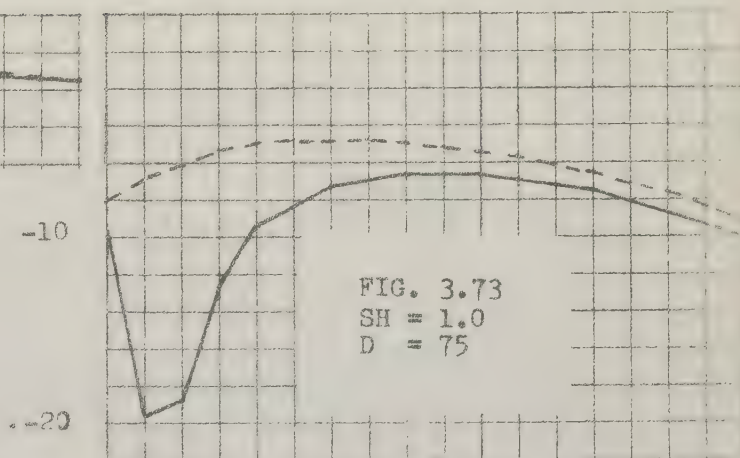
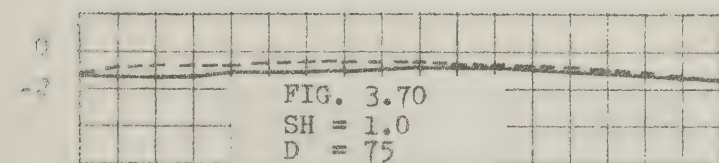
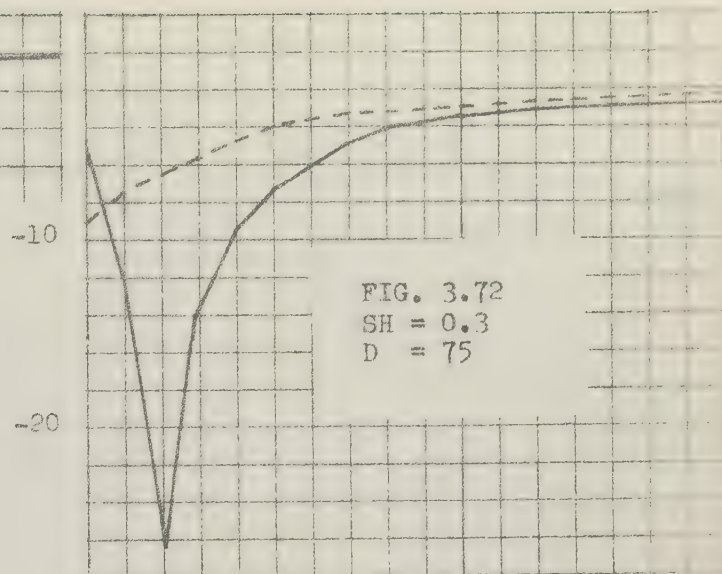
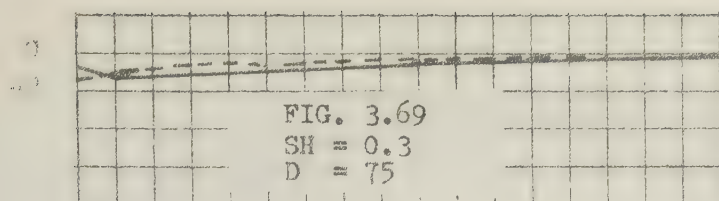
As the theoretical expressions in Section 2.2 are very difficult to penetrate some numerical comparisons between cylindrical and spherical waves have been made. The cylindrical computations are based on Eqs. (2.4), (2.34), (2.35) and (2.37)-(2.39). The ground surface has been assumed to be acoustically equivalent to measurement site 2.

The calculated examples in Figs. 3.69-3.80 indicate that the difference between cylindrical and spherical waves can be considerable. As anticipated this is especially the case for low receiver heights, that is when the deviation from the plane wave reflection coefficient is generally largest. In these cases it is obvious that the separated solutions as given by Eqs. (2.131) and (2.134) will break down providing that the diffracted wave is of cylindrical character, that is when  $r_0 \gg r$  and  $\sigma \gg 1$ .

At 125 Hz the difference between the two wave-types is insignificant because the ground surface is almost totally reflecting.







FIGS. 3.69 - 3.71

125 Hz

$$\underline{u} = 0.005 - i \cdot 0.005$$

— spherical - - cylindrical

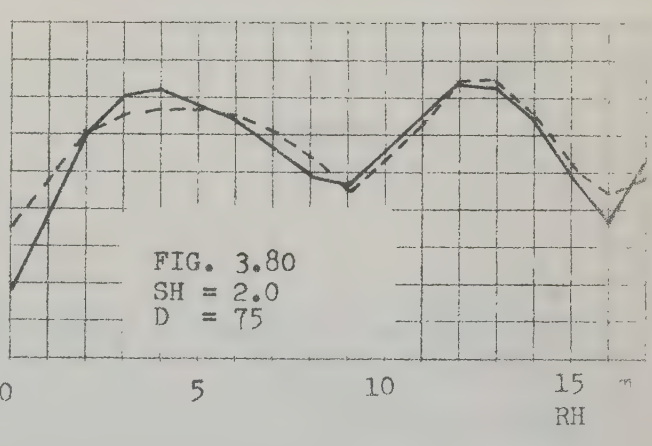
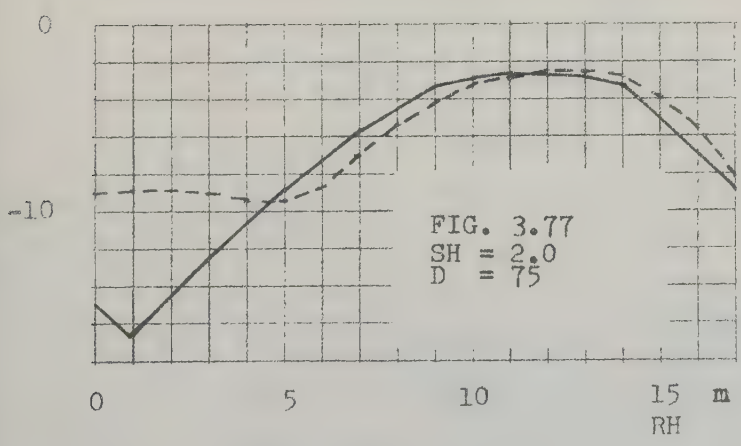
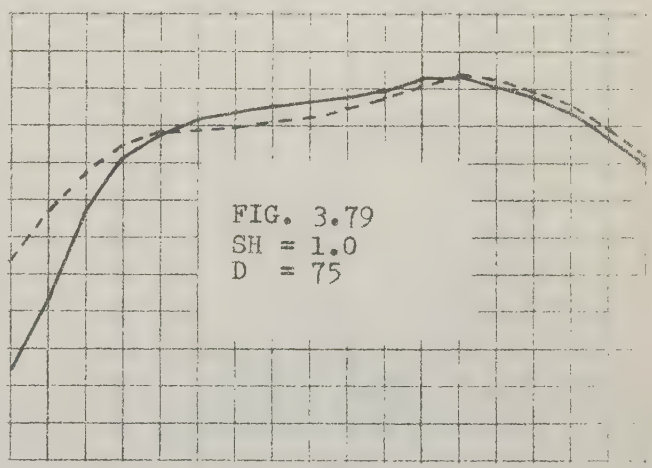
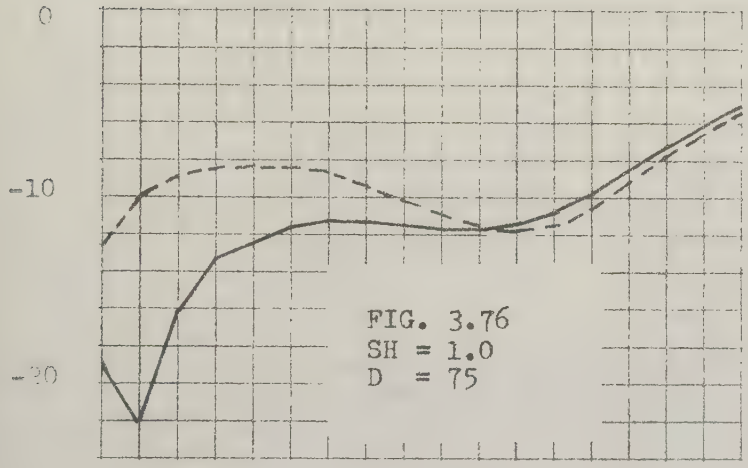
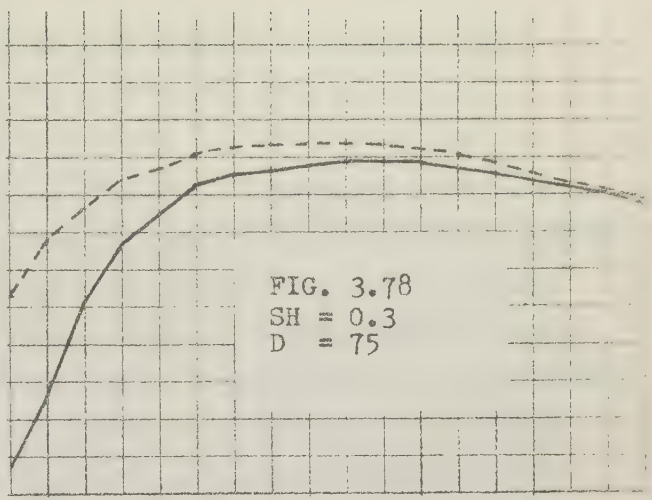
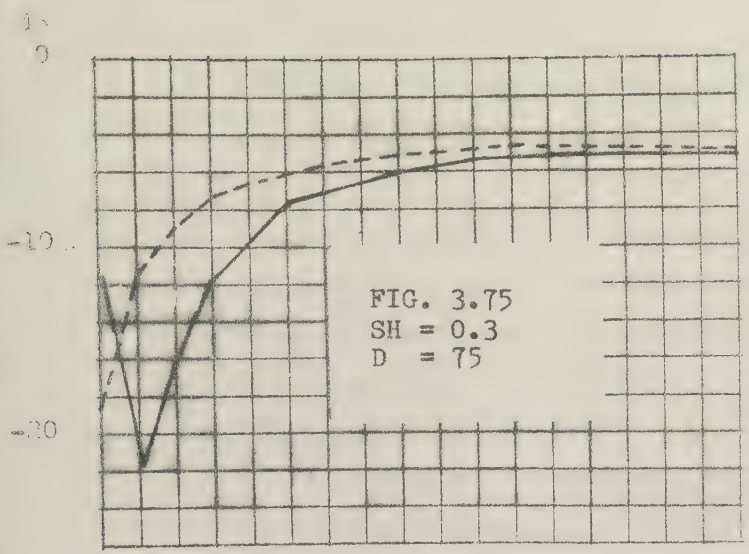
FIGS. 3.72 - 3.74

250 Hz

$$\underline{u} = 0.05 - i \cdot 0.05$$

— spherical - - cylindrical





FIGS. 3.75 - 3.77

500 Hz

$$\underline{v} = 0.06 - i \cdot 0.06$$

— spherical  
-- cylindrical

FIGS. 3.78 - 3.80

1000 Hz

$$\underline{v} = 0.10 - i \cdot 0.05$$

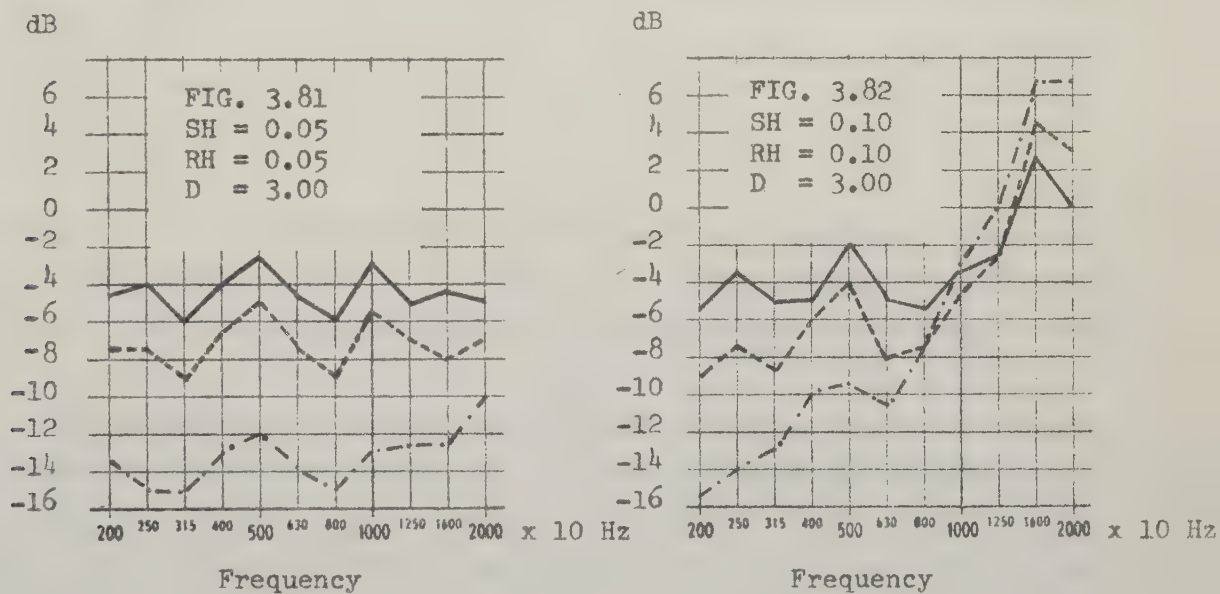
— spherical  
-- cylindrical





3.3. The active width of the reflecting ground strip between source and receiver. Model measurements.

The model measurements were performed inside a large sound absorbing cage. The reflecting surface consisted of two 0.013 m thick 0.6 x 1.5 m<sup>2</sup> large gypsum boards which were placed d m apart on 0.1 m high studs. The space between the boards was filled with 1.0 m thick mineral wool batts. The distance between the source, a 2" tweeter, and the receiver, a 1/2" condenser microphone, was 3.0 m and the measurements were made in 1/3-octave bands. The distance d, which was symmetric around the half-way line between the source and the receiver, was in the range 0-2 m. The reference sound pressure level is the one that was obtained for d = 0.



FIGS. 3.81 - 3.82

— d = 0.5      --- d = 1.0      . - . - d = 2.0

Fig. 3.81 indicates that when both source and receiver are situated close to the ground the active width of the reflecting strip can be very large. In this case we have a considerable change in level when d is increased from 1.0 to 2.0 m even at 20 kHz. This means that the active width is at least of the order of magnitude 100 wavelengths. When both the height of the source and the receiver are doubled we see from Fig. 3.82 that the difference between the level curves becomes small at 8 kHz, that is the active width in this case is about 0.5 m or 10 wavelengths.



Even if these very few measurements only can give a very rough indication about what happens it still seems evident that very large ground areas must be considered near grazing incidence.

### 3.4. Comparisons between the "exact" and the approximate solution for the diffracted reflected spherical wave.

In Section 2.3.3 the diffracted reflected field was given by Eq. (2.123), the "exact" solution, and Eq. (2.127), which will be called the receiver approximation as the distance that is used in this case to compute the reflection coefficient is the one between the source and the receiver. If we instead use the distance between the source and the top of the barrier we will get a solution which can be called the barrier approximation. This kind of solution is of particular interest because if it is valid it will in many cases be possible to separate the ground attenuation at the source side of the barrier from all other effects.

In Figs. 3.83 - 3.90 the results of some numerical computations are shown. For the approximate solutions the reference values have been the values of the "exact" solution. Negative receiver heights indicate the image receiver.

With a few exceptions the differences between the three types of solutions are rather small. In most cases the receiver approximation as given by Eq. (2.127) is the best approximation, which it of course should be. The barrier approximation is however surprisingly good and this is due to the fact that in the calculated cases the G-function is of less importance for the result.

In Section 3.5.2 additional comparisons are made between "exact" and approximate solutions.



BARRIER HEIGHTS  
ADJUSTANCE 1.00000000  
WAVELENGTH 1.85000000

FIG. 3.83

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT<br>LEVEL | BARRIER APPROX<br>LEVEL | RECEIVED APPROX<br>LEVEL | PHASE |
|--------|----------|----------|----------------|-------------------------|--------------------------|-------|
| .3     | 20+ 20   | 3.5      | -25.2          | 1.9                     | 1.4                      | -1.1  |
|        |          | 0.0      | -31.0          | 1.9                     | 1.9                      | 1.1   |
|        |          | -3.5     | -36.6          | 1.9                     | 1.1                      | -1.1  |
| 1.0    | 20+ 20   | 3.5      | -24.7          | 0                       | 2.8                      | 1.1   |
|        |          | 0.0      | -30.3          | 0                       | 1.9                      | 1.1   |
|        |          | -3.5     | -35.7          | 1.5                     | 1.1                      | -1.1  |
| 2.0    | 20+ 20   | 3.5      | -24.7          | 4                       | 2.0                      | 1.1   |
|        |          | 0.0      | -29.7          | 2                       | 1.3                      | 1.1   |
|        |          | -3.5     | -32.8          | 1.1                     | 1.0                      | -1.1  |
| .3     | 20+ 40   | 3.5      | -27.0          | 1.8                     | 1.2                      | 1.1   |
|        |          | 0.0      | -30.2          | 1.1                     | 1.1                      | 1.1   |
|        |          | -3.5     | -32.7          | 1.1                     | 1.2                      | -1.1  |
| 1.0    | 20+ 40   | 3.5      | -26.5          | 1.7                     | 3.3                      | 1.1   |
|        |          | 0.0      | -29.6          | 1.6                     | 2.5                      | 1.1   |
|        |          | -3.5     | -31.9          | 1.9                     | 1.1                      | -1.1  |
| 2.0    | 20+ 40   | 3.5      | -26.5          | 4                       | 2.2                      | 1.1   |
|        |          | 0.0      | -29.2          | 1.6                     | 1.6                      | 1.1   |
|        |          | -3.5     | -31.3          | 1.3                     | 1.1                      | -1.1  |
| .3     | 20+ 60   | 3.5      | -27.7          | 1.8                     | 3.1                      | 1.1   |
|        |          | 0.0      | -30.0          | 1.3                     | 1.6                      | 1.1   |
|        |          | -3.5     | -31.8          | 1.0                     | 2.4                      | -1.1  |
| 1.0    | 20+ 60   | 3.5      | -27.3          | 1.9                     | 3.9                      | 1.1   |
|        |          | 0.0      | -29.4          | 1.6                     | 2.8                      | 1.1   |
|        |          | -3.5     | -31.1          | 1.5                     | 2.3                      | -1.1  |
| 2.0    | 20+ 60   | 3.5      | -27.2          | 4                       | 2.2                      | 1.1   |
|        |          | 0.0      | -29.1          | 1.6                     | 1.6                      | 1.1   |
|        |          | -3.5     | -30.6          | 1.2                     | 1.5                      | -1.1  |

N= 10  
BARRIER HEIGHTS 1.00000000  
ADJUSTANCE 1.00000000  
WAVELENGTH 1.85000000

FIG. 3.84

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT<br>LEVEL | BARRIER APPROX<br>LEVEL | RECEIVED APPROX<br>LEVEL | PHASE |
|--------|----------|----------|----------------|-------------------------|--------------------------|-------|
| .3     | 20+ 20   | 3.5      | -19.2          | 1.9                     | 1.4                      | -1.1  |
|        |          | 0.0      | -24.8          | 1.9                     | 1.9                      | 1.1   |
|        |          | -3.5     | -30.4          | 1.9                     | 1.1                      | -1.1  |
| 1.0    | 20+ 20   | 3.5      | -19.0          | 1.5                     | 1.4                      | 1.1   |
|        |          | 0.0      | -24.1          | 1.3                     | 1.6                      | 1.1   |
|        |          | -3.5     | -29.4          | 1.6                     | 1.1                      | -1.1  |
| 2.0    | 20+ 20   | 3.5      | -19.4          | 1.7                     | 1.4                      | 1.1   |
|        |          | 0.0      | -24.1          | 1.3                     | 1.6                      | 1.1   |
|        |          | -3.5     | -29.6          | 1.5                     | 1.2                      | -1.1  |
| .3     | 20+ 40   | 3.5      | -21.8          | 1.7                     | 1.6                      | 1.1   |
|        |          | 0.0      | -27.6          | 1.1                     | 1.7                      | 1.1   |
|        |          | -3.5     | -31.1          | 1.8                     | 1.2                      | -1.1  |
| 1.0    | 20+ 40   | 3.5      | -25.4          | 1.4                     | 1.2                      | 1.1   |
|        |          | 0.0      | -30.2          | 1.3                     | 1.3                      | 1.1   |
|        |          | -3.5     | -33.5          | 1.9                     | 1.2                      | -1.1  |
| 2.0    | 20+ 40   | 3.5      | -27.9          | 1.8                     | 1.2                      | 1.1   |
|        |          | 0.0      | -31.8          | 1.1                     | 1.3                      | 1.1   |
|        |          | -3.5     | -34.8          | 1.8                     | 1.2                      | -1.1  |
| .3     | 20+ 60   | 3.5      | -23.0          | 1.4                     | 1.2                      | 1.1   |
|        |          | 0.0      | -27.1          | 1.2                     | 1.2                      | 1.1   |
|        |          | -3.5     | -29.9          | 1.6                     | 1.2                      | -1.1  |
| 1.0    | 20+ 60   | 3.5      | -26.5          | 1.9                     | 1.2                      | 1.1   |
|        |          | 0.0      | -29.9          | 1.4                     | 1.3                      | 1.1   |
|        |          | -3.5     | -32.4          | 1.1                     | 1.2                      | -1.1  |
| 2.0    | 20+ 60   | 3.5      | -27.0          | 1.8                     | 1.2                      | 1.1   |
|        |          | 0.0      | -29.4          | 1.3                     | 1.3                      | 1.1   |
|        |          | -3.5     | -31.3          | 1.3                     | 1.2                      | -1.1  |

N= 10  
BARRIER HEIGHTS 1.00000000  
ADJUSTANCE 1.00000000  
WAVELENGTH 1.85000000

FIG. 3.85

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT<br>LEVEL | BARRIER APPROX<br>LEVEL | RECEIVED APPROX<br>LEVEL | PHASE |
|--------|----------|----------|----------------|-------------------------|--------------------------|-------|
| .3     | 20+ 20   | 3.5      | -24.7          | 1.9                     | 1.4                      | -1.1  |
|        |          | 0.0      | -31.0          | 1.9                     | 1.9                      | 1.1   |
|        |          | -3.5     | -36.6          | 1.9                     | 1.1                      | -1.1  |
| 1.0    | 20+ 20   | 3.5      | -24.7          | 0                       | 2.8                      | 1.1   |
|        |          | 0.0      | -30.3          | 0                       | 1.9                      | 1.1   |
|        |          | -3.5     | -35.7          | 1.5                     | 1.1                      | -1.1  |
| 2.0    | 20+ 20   | 3.5      | -24.7          | 4                       | 2.0                      | 1.1   |
|        |          | 0.0      | -29.7          | 2                       | 1.3                      | 1.1   |
|        |          | -3.5     | -32.8          | 1.1                     | 1.0                      | -1.1  |
| .3     | 20+ 40   | 3.5      | -27.0          | 1.8                     | 1.2                      | 1.1   |
|        |          | 0.0      | -30.2          | 1.1                     | 1.1                      | 1.1   |
|        |          | -3.5     | -32.7          | 1.1                     | 1.2                      | -1.1  |
| 1.0    | 20+ 40   | 3.5      | -26.5          | 1.7                     | 3.3                      | 1.1   |
|        |          | 0.0      | -29.6          | 1.6                     | 2.5                      | 1.1   |
|        |          | -3.5     | -31.9          | 1.9                     | 1.1                      | -1.1  |
| 2.0    | 20+ 40   | 3.5      | -26.5          | 4                       | 2.2                      | 1.1   |
|        |          | 0.0      | -29.2          | 1.6                     | 1.6                      | 1.1   |
|        |          | -3.5     | -31.3          | 1.3                     | 1.1                      | -1.1  |
| .3     | 20+ 60   | 3.5      | -27.7          | 1.8                     | 3.1                      | 1.1   |
|        |          | 0.0      | -30.0          | 1.3                     | 1.6                      | 1.1   |
|        |          | -3.5     | -31.8          | 1.0                     | 2.4                      | -1.1  |
| 1.0    | 20+ 60   | 3.5      | -27.3          | 1.9                     | 3.9                      | 1.1   |
|        |          | 0.0      | -29.4          | 1.6                     | 2.8                      | 1.1   |
|        |          | -3.5     | -31.1          | 1.5                     | 2.3                      | -1.1  |
| 2.0    | 20+ 60   | 3.5      | -27.2          | 4                       | 2.2                      | 1.1   |
|        |          | 0.0      | -29.1          | 1.6                     | 1.6                      | 1.1   |
|        |          | -3.5     | -30.6          | 1.2                     | 1.5                      | -1.1  |

N= 10  
BARRIER HEIGHTS 1.00000000  
ADJUSTANCE 1.00000000  
WAVELENGTH 1.85000000

FIG. 3.86

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT<br>LEVEL | BARRIER APPROX<br>LEVEL | RECEIVED APPROX<br>LEVEL | PHASE |
|--------|----------|----------|----------------|-------------------------|--------------------------|-------|
| .3     | 20+ 20   | 3.5      | -24.7          | 1.9                     | 1.4                      | -1.1  |
|        |          | 0.0      | -31.0          | 1.9                     | 1.9                      | 1.1   |
|        |          | -3.5     | -36.6          | 1.9                     | 1.1                      | -1.1  |
| 1.0    | 20+ 20   | 3.5      | -24.7          | 0                       | 2.8                      | 1.1   |
|        |          | 0.0      | -30.3          | 0                       | 1.9                      | 1.1   |
|        |          | -3.5     | -35.7          | 1.5                     | 1.1                      | -1.1  |
| 2.0    | 20+ 20   | 3.5      | -24.7          | 4                       | 2.0                      | 1.1   |
|        |          | 0.0      | -29.7          | 2                       | 1.3                      | 1.1   |
|        |          | -3.5     | -32.8          | 1.1                     | 1.0                      | -1.1  |
| .3     | 20+ 40   | 3.5      | -27.0          | 1.8                     | 1.2                      | 1.1   |
|        |          | 0.0      | -30.2          | 1.1                     | 1.1                      | 1.1   |
|        |          | -3.5     | -32.7          | 1.1                     | 1.2                      | -1.1  |
| 1.0    | 20+ 40   | 3.5      | -26.5          | 1.7                     | 3.3                      | 1.1   |
|        |          | 0.0      | -29.6          | 1.6                     | 2.5                      | 1.1   |
|        |          | -3.5     | -31.9          | 1.9                     | 1.1                      | -1.1  |
| 2.0    | 20+ 40   | 3.5      | -26.5          | 4                       | 2.2                      | 1.1   |
|        |          | 0.0      | -29.2          | 1.6                     | 1.6                      | 1.1   |
|        |          | -3.5     | -31.3          | 1.3                     | 1.1                      | -1.1  |
| .3     | 20+ 60   | 3.5      | -27.7          | 1.8                     | 3.1                      | 1.1   |
|        |          | 0.0      | -30.0          | 1.3                     | 1.6                      | 1.1   |
|        |          | -3.5     | -31.8          | 1.0                     | 2.4                      | -1.1  |
| 1.0    | 20+ 60   | 3.5      | -27.3          | 1.9                     | 3.9                      | 1.1   |
|        |          | 0.0      | -29.4          | 1.6                     | 2.8                      | 1.1   |
|        |          | -3.5     | -31.1          | 1.5                     | 2.3                      | -1.1  |
| 2.0    | 20+ 60   | 3.5      | -27.2          | 4                       | 2.2                      | 1.1   |
|        |          | 0.0      | -29.1          | 1.6                     | 1.6                      | 1.1   |
|        |          | -3.5     | -30.6          | 1.2                     | 1.5                      | -1.1  |

N= 10  
BARRIER HEIGHTS 1.00000000  
ADJUSTANCE 1.00000000  
WAVELENGTH 1.85000000





FIG. 3.85

NE 10  
BARRIER HEIGHT= 2.8000+00  
ADMITTANCE= 5.0000+02  
WAVENUMBER= 4.5700+00

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT |        | BARRIER APPROX |       | REFLECTER APPROX |       |
|--------|----------|----------|-------|--------|----------------|-------|------------------|-------|
|        |          |          | LEVEL | PHASE  | LEVEL          | PHASE | LEVEL            | PHASE |
| .3     | 20+ 20   | 3.5      | -16.4 | -267.6 | .8             | 2.2   | 2.0              | -53.1 |
|        |          | 0.0      | -15.7 | -162.0 | .6             | 5.4   | .7               | -3.6  |
|        |          | -3.5     | -22.7 | 12.3   | -1.2           | 0.5   | .1               | -3.0  |
| 1.0    | 20+ 20   | 3.5      | -15.9 | -266.6 | .7             | 3.1   | 1.1              | -2.0  |
|        |          | 0.0      | -19.7 | -145.1 | .4             | 4.2   | .4               | 1.8   |
|        |          | -3.5     | -22.6 | 44.8   | -.4            | 4.3   | .1               | -1.7  |
| 2.0    | 20+ 20   | 3.5      | -16.1 | -250.5 | .5             | 2.1   | .5               | -1.7  |
|        |          | 0.0      | -19.9 | -106.3 | .2             | 2.3   | .2               | -.6   |
|        |          | -3.5     | -22.7 | -95.2  | .0             | 2.1   | .0               | -.7   |
| .3     | 20+ 40   | 3.5      | -16.9 | -252.5 | .1             | 5.6   | 1.4              | -5.9  |
|        |          | 0.0      | -18.9 | -101.6 | -.7            | 8.0   | .6               | -3.6  |
|        |          | -3.5     | -20.7 | -106.6 | -1.2           | 8.9   | .1               | -3.4  |
| 1.0    | 20+ 40   | 3.5      | -16.9 | -244.4 | .4             | 5.4   | .8               | -2.2  |
|        |          | 0.0      | -19.1 | -12.7  | .4             | 9.1   | .4               | -1.6  |
|        |          | -3.5     | -20.9 | -77.2  | -.3            | 9.3   | .0               | -1.4  |
| 2.0    | 20+ 40   | 3.5      | -17.4 | -216.4 | .5             | 2.7   | .4               | -.6   |
|        |          | 0.0      | -19.6 | -130.5 | .3             | 2.7   | .2               | -.6   |
|        |          | -3.5     | -21.4 | -20.3  | .2             | 2.8   | .1               | -.4   |
| .3     | 20+ 60   | 3.5      | -17.2 | -246.2 | -.1            | 8.3   | 1.0              | -5.5  |
|        |          | 0.0      | -18.6 | -202.9 | -.7            | 10.0  | .6               | -3.8  |
|        |          | -3.5     | -19.9 | -147.3 | -1.1           | 10.8  | .1               | -3.3  |
| 1.0    | 20+ 60   | 3.5      | -17.5 | -233.7 | .4             | 6.7   | .7               | -1.3  |
|        |          | 0.0      | -19.0 | -182.2 | .1             | 7.1   | .3               | -1.0  |
|        |          | -3.5     | -20.3 | -118.7 | -.1            | 7.2   | -.3              | -.3   |
| 2.0    | 20+ 60   | 3.5      | -18.1 | -199.8 | .6             | 3.3   | .5               | -.3   |
|        |          | 0.0      | -18.6 | -137.6 | .5             | 3.7   | .3               | -.4   |
|        |          | -3.5     | -20.9 | -63.2  | .5             | 3.8   | .4               | -.5   |

FIG. 3.90

NE 10

|                 |           |
|-----------------|-----------|
| BARRIER HEIGHT= | 2.8000+00 |
| ADMITTANCE      | 5.0000+02 |
| WAVENUMBER      | 4.5700+00 |

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT  |        | BARRIER APPROX |       | REFLECTER APPROX |       |
|--------|----------|----------|--------|--------|----------------|-------|------------------|-------|
|        |          |          | LEVEL  | PHASE  | LEVEL          | PHASE | LEVEL            | PHASE |
| .3     | 40+ 20   | 3.5      | -15.7  | -208.4 | 1.5            | -17.7 | 2.0              | -33.8 |
|        |          | 0.0      | -120.7 | .6     | 2.3            | 1.6   | -13.9            |       |
|        |          | -3.5     | -23.8  | 54.7   | -.5            | 8.0   | .6               | -2.8  |
| 1.0    | 40+ 20   | 3.5      | -16.9  | -237.5 | 2.2            | -9.2  | 1.0              | -20.3 |
|        |          | 0.0      | -21.1  | -134.9 | .7             | 3.1   | 1.5              | -6.9  |
|        |          | -3.5     | -24.2  | 54.9   | -1.2           | 8.4   | .6               | -5.2  |
| 2.0    | 40+ 20   | 3.5      | -16.6  | -261.2 | 1.9            | -1.2  | 2.2              | -8.4  |
|        |          | 0.0      | -21.6  | -193.7 | .7             | 3.0   | 1.0              | -3.8  |
|        |          | -3.5     | -24.2  | 67.6   | .1             | 3.6   | .4               | -3.0  |
| .3     | 40+ 40   | 3.5      | -16.2  | -203.4 | .9             | -10.7 | 2.1              | -35.3 |
|        |          | 0.0      | -19.2  | -157.7 | .2             | 4.7   | 1.4              | -21.1 |
|        |          | -3.5     | -21.3  | -71.2  | -.7            | 12.0  | .5               | -12.8 |
| 1.0    | 40+ 40   | 3.5      | -17.6  | -227.2 | 1.7            | -4.3  | 2.8              | -21.8 |
|        |          | 0.0      | -20.0  | -169.1 | .5             | 5.0   | 1.4              | -13.5 |
|        |          | -3.5     | -22.1  | -71.9  | -.3            | 8.9   | .5               | -8.7  |
| 2.0    | 40+ 40   | 3.5      | -17.7  | -243.6 | 1.6            | 1.8   | 1.0              | -8.2  |
|        |          | 0.0      | -20.2  | -170.2 | .7             | 5.2   | 1.0              | -5.4  |
|        |          | -3.5     | -22.4  | -60.0  | .1             | 6.4   | .4               | -4.3  |
| .3     | 40+ 60   | 3.5      | -16.4  | -202.4 | .5             | -5.0  | 1.6              | -34.7 |
|        |          | 0.0      | -18.5  | -170.1 | -.1            | 7.2   | 1.0              | -22.4 |
|        |          | -3.5     | -20.1  | -115.1 | -.8            | 14.3  | .3               | -15.6 |
| 1.0    | 40+ 60   | 3.5      | -17.9  | -222.8 | 1.8            | -.2   | 2.1              | -21.2 |
|        |          | 0.0      | -19.6  | -180.4 | .7             | 7.1   | 1.4              | -13.2 |
|        |          | -3.5     | -21.1  | -116.5 | -.1            | 10.0  | .5               | -10.1 |
| 2.0    | 40+ 60   | 3.5      | -18.2  | -233.6 | 1.0            | 6.2   | 1.6              | -20.1 |
|        |          | 0.0      | -20.0  | -179.0 | .8             | 6.2   | 1.6              | -17.9 |
|        |          | -3.5     | -21.6  | -104.4 | .4             | 7.9   | .4               | -10.4 |

FIG. 3.87

NE 10  
BARRIER HEIGHT= 2.8000+00  
ADMITTANCE= 5.0000+02  
WAVENUMBER= 9.1500+00

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT |        | BARRIER APPROX |       | REFLECTER APPROX |       |
|--------|----------|----------|-------|--------|----------------|-------|------------------|-------|
|        |          |          | LEVEL | PHASE  | LEVEL          | PHASE | LEVEL            | PHASE |
| .3     | 20+ 20   | 3.5      | -15.4 | -204.4 | .3             | 6.9   | .1               | -1.1  |
|        |          | 0.0      | -20.4 | -70.8  | .1             | 2.6   | .8               | -3.4  |
|        |          | -3.5     | -24.0 | -93.5  | -.3            | 2.0   | -.7              | -3.0  |
| 1.0    | 20+ 20   | 3.5      | -13.0 | -253.4 | .2             | 1.9   | .9               | -.9   |
|        |          | 0.0      | -20.5 | -23.3  | .0             | 1.0   | .3               | -.3   |
|        |          | -3.5     | -20.1 | -6.8   | -.1            | 1.2   | -.6              | -.6   |
| 2.0    | 20+ 20   | 3.5      | -16.7 | -209.0 | .2             | 1.6   | .2               | -.2   |
|        |          | 0.0      | -21.4 | -105.3 | .1             | .5    | .0               | -.3   |
|        |          | -3.5     | -22.0 | -35.3  | .1             | .5    | .0               | -.3   |
| .3     | 20+ 40   | 3.5      | -16.8 | -247.7 | .1             | 3.4   | .1               | -.1   |
|        |          | 0.0      | -22.4 | -10.8  | -.1            | 3.1   | -.4              | -.4   |
|        |          | -3.5     | -24.3 | -74.3  | .1             | 1.7   | .8               | -3.4  |
| 1.0    | 20+ 40   | 3.5      | -17.1 | -241.5 | .1             | 2.7   | .1               | -.1   |
|        |          | 0.0      | -20.3 | -74.3  | .1             | 3.2   | .1               | -.1   |
|        |          | -3.5     | -24.2 | -40.9  | -.1            | 1.7   | -.2              | -.2   |
| 2.0    | 20+ 40   | 3.5      | -18.3 | -243.9 | .1             | 1.3   | .1               | -.1   |
|        |          | 0.0      | -22.3 | -121.0 | .1             | .9    | .0               | -.0   |
|        |          | -3.5     | -23.0 | -23.0  | .1             | .9    | .0               | -.0   |
| .3     | 20+ 60   | 3.5      | -17.2 | -231.8 | .3             | 3.8   | .5               | -.0   |
|        |          | 0.0      | -19.4 | -147.3 | .7             | 3.3   | .3               | -.5   |
|        |          | -3.5     | -21.2 | -38.5  | .1             | 3.3   | .5               | -.5   |
| 1.0    | 20+ 60   | 3.5      | -17.0 | -188.9 | .2             | 4.5   | .4               | -.4   |
|        |          | 0.0      | -19.9 | -99.0  | .1             | 2.1   | .1               | -.1   |
|        |          | -3.5     | -21.6 | -33.0  | .1             | 2.1   | .0               | -.0   |
| 2.0    | 20+ 60   | 3.5      | -19.0 | -111.1 | .2             | 1.3   | .2               | -.2   |
|        |          | 0.0      | -20.0 | -9.3   | .2             | 1.3   | .3               | -.3   |
|        |          | -3.5     | -22.3 | -204.8 | .1             | 1.0   | -.0              | -.0   |

FIG. 3.88

NE 10  
BARRIER HEIGHT= 2.8000+00  
ADMITTANCE= 5.0000+02  
WAVENUMBER= 9.1500+00

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT |        | BARRIER APPROX |       | REFLECTER APPROX |       |
|--------|----------|----------|-------|--------|----------------|-------|------------------|-------|
|        |          |          | LEVEL | PHASE  | LEVEL          | PHASE | LEVEL            | PHASE |
| .3     | 40+ 20   | 3.5      | -17.4 | -67.8  | 1.0            | 2.9   | 2.9              | -3.8  |
|        |          | 0.0      | -22.9 | -112.9 | .4             | 4.5   | .7               | -1.0  |
|        |          | -3.5     | -26.9 | -105.6 | -.1            | 3.7   | .1               | -1.0  |
| 1.0    | 40+ 20   | 3.5      | -13.7 | -63.0  | 1.0            | 2.6   | .9               | -.9   |
|        |          | 0.0      | -22.0 | -97.2  | .1             | 3.1   | .4               | -.4   |
|        |          | -3.5     | -26.1 | -69.3  | -.4            | 2.4   | .1               | -.2   |
| 2.0    | 40+ 20   | 3.5      | -15.1 | -69.9  | .7             | 2.0   | .2               | -.2   |
|        |          | 0.0      | -21.6 | -63.2  | .3             | 3.2   | .4               | -.4   |
|        |          | -3.5     | -25.6 | -5.8   | -.0            | 2.1   | .7               | -.7   |
| .3     | 40+ 40   | 3.5      | -18.1 | -84.9  | .8             | 4.1   | 4.1              | -4.8  |
|        |          | 0.0      | -21.3 | -160.2 | .6             | 6.4   | .3               | -.3   |
|        |          | -3.5     | -24.2 | -11.3  | -.1            | 6.0   | .0               | -.0   |
| 1.0    | 40+ 40   | 3.5      | -17.1 | -86.6  | .7             | 3.5   | .7               | -.7   |
|        |          | 0.0      | -20.9 | -150.8 | .1             | 5.0   | .1               | -.1   |
|        |          | -3.5     | -23.7 | -44.2  | -.4            | 4.7   | .6               | -.6   |
| 2.0    | 40+ 40   | 3.5      | -17.0 | -255.1 | .6             | 3.0   | .1               | -.1   |
|        |          | 0.0      | -20.4 | -111.6 | .3             | 3.3   | .4               | -.4   |
|        |          | -3.5     | -23.7 | -255.0 | .1             | 3.0   | .1               | -.0   |
| .3     | 40+ 60   | 3.5      | -18.4 | -266.0 | .4             | 8.2   | 1.9              | -55.5 |
|        |          | 0.0      | -20.8 | -180.0 | .7             | 8.3   | .8               | -.6   |
|        |          | -3.5     | -23.7 | -71.1  | -.3            | 8.3   | .2               | -2.4  |
| 1.0    | 40+ 60   | 3.5      | -17.7 | -259.3 | .5             | 5.5   | 1.0              | -1.2  |
|        |          | 0.0      | -20.3 | -168.9 | .3             | 5.7   | .4               | -.9   |
|        |          | -3.5     | -23.7 | -39.5  | -.4            | 6.2   | .1               | -.7   |
| 2.0    | 40+ 60   | 3.5      | -17.0 | -233.8 | .0             | 3.4   | .2               | -.2   |
|        |          | 0.0      | -20.6 | -106.8 | .3             | 3.1   | .1               | -.4   |
|        |          | -3.5     | -23.8 | -20.2  | .2             | 3.1   | .1               | -.4   |



FIG. 3.91

NE 15  
BARRIER HEIGHT= 2-8000+00  
ADMITTANCE= 3-0000+02  
RAVENNUMBER= 4-5700+00

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT |        | BARRIER APPROX |       | RECEIVED APPROX |       |
|--------|----------|----------|-------|--------|----------------|-------|-----------------|-------|
|        |          |          | LEVEL | PHASE  | LEVEL          | PHASE | LEVEL           | PHASE |
| .3     | 20+ 20   | 3-5      | -14.3 | 58.4   | -1.1           | -4    | 1.0             | .4    |
|        |          | 0-0      | -18.3 | -102.2 | -1.1           | -3    | .4              | .1    |
|        |          | -3-5     | -21.4 | -16.7  | -1.1           | -3    | .0              | -1.1  |
| 1.0    | 20+ 20   | 3-5      | -14.1 | 69.4   | .0             | 1.9   | .6              | .2    |
|        |          | 0-0      | -19.1 | -167.4 | -4             | 1.6   | .2              | -1.1  |
|        |          | -3-5     | -21.3 | 23.0   | -5             | 1.3   | .0              | -1.4  |
| 2.0    | 20+ 20   | 3-5      | -14.6 | -267.1 | .2             | 1.7   | .4              | .2    |
|        |          | 0-0      | -18.6 | -124.4 | -0             | 1.4   | .1              | -1.1  |
|        |          | -3-5     | -21.5 | 87.7   | -1             | 1.2   | .0              | -1.2  |
| .3     | 20+ 40   | 3-5      | -14.9 | 76.1   | -6             | 1.2   | .7              | .2    |
|        |          | 0-0      | -17.2 | -220.6 | -1.1           | 1.2   | .3              | -1.2  |
|        |          | -3-5     | -19.3 | -134.5 | -1.3           | 1.0   | .0              | -1.4  |
| 1.0    | 20+ 40   | 3-5      | -15.2 | -266.5 | -2             | 2.7   | .4              | .0    |
|        |          | 0-0      | -17.5 | -193.9 | -4             | 2.5   | .2              | -1.2  |
|        |          | -3-5     | -19.5 | -98.0  | -6             | 2.3   | .0              | -1.4  |
| 2.0    | 20+ 40   | 3-5      | -16.0 | -233.2 | .2             | 2.6   | .2              | .5    |
|        |          | 0-0      | -18.3 | -117.1 | .1             | 2.3   | .1              | .3    |
|        |          | -3-5     | -20.1 | -37.0  | .0             | 2.0   | .6              | -1.6  |
| .3     | 20+ 60   | 3-5      | -15.2 | 84.3   | -0             | 2.0   | .5              | .8    |
|        |          | 0-0      | -16.0 | -230.8 | -1.1           | 2.1   | .2              | -1.7  |
|        |          | -3-5     | -18.3 | -174.3 | -1.4           | 2.0   | .0              | -1.8  |
| 1.0    | 20+ 60   | 3-5      | -15.6 | -285.0 | -3             | 3.2   | .2              | .0    |
|        |          | 0-0      | -17.2 | -203.0 | -5             | 3.1   | .1              | -1.1  |
|        |          | -3-5     | -18.7 | -139.2 | -6             | 2.9   | .0              | -1.3  |
| 2.0    | 20+ 60   | 3-5      | -16.4 | -215.1 | .0             | 1.7   | .0              | .6    |
|        |          | 0-0      | -18.0 | -152.8 | -1             | 1.5   | .0              | -1.8  |
|        |          | -3-5     | -19.4 | -78.1  | -1.0           | 1.6   | .0              | -1.4  |

FIG. 3.93

NE 15  
BARRIER HEIGHT= 2-8000+00  
ADMITTANCE= 3-0000+03  
RAVENNUMBER= 2-2800+00

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT |        | BARRIER APPROX |       | RECEIVED APPROX |       |
|--------|----------|----------|-------|--------|----------------|-------|-----------------|-------|
|        |          |          | LEVEL | PHASE  | LEVEL          | PHASE | LEVEL           | PHASE |
| .3     | 20+ 20   | 3-5      | -9.7  | 28.9   | .1             | -1.3  | .1              | .2    |
|        |          | 0-0      | -13.2 | 89.5   | .0             | -1.3  | .1              | .1    |
|        |          | -3-5     | -16.0 | -178.3 | -2             | -1.6  | .0              | -1.1  |
| 1.0    | 20+ 20   | 3-5      | -10.2 | 37.7   | .0             | -7    | .1              | .2    |
|        |          | 0-0      | -13.6 | -255.0 | -1             | -8    | .0              | -1.1  |
|        |          | -3-5     | -16.4 | -150.3 | -3             | -9    | .0              | -1.1  |
| 2.0    | 20+ 20   | 3-5      | -11.0 | 52.4   | .0             | -1    | .1              | .0    |
|        |          | 0-0      | -14.3 | -230.9 | -1             | -2    | .0              | -1.1  |
|        |          | -3-5     | -17.0 | -121.8 | -1             | -4    | .0              | -1.3  |
| .3     | 20+ 40   | 3-5      | -10.4 | 99.9   | -1             | -1.7  | .1              | .2    |
|        |          | 0-0      | -12.4 | 75.0   | -2             | -1.6  | .0              | .3    |
|        |          | -3-5     | -14.2 | -239.1 | -3             | -1.6  | .0              | .3    |
| 1.0    | 20+ 40   | 3-5      | -11.1 | 51.7   | -1             | -7    | .1              | .0    |
|        |          | 0-0      | -13.0 | -268.7 | -2             | -8    | .0              | -1.0  |
|        |          | -3-5     | -14.8 | -218.2 | -2             | -8    | .0              | -1.0  |
| 2.0    | 20+ 40   | 3-5      | -12.0 | 71.5   | .0             | -0    | .0              | .0    |
|        |          | 0-0      | -13.9 | -242.5 | -1             | -0    | .0              | -1.1  |
|        |          | -3-5     | -15.6 | -185.3 | -2             | -1    | .0              | -1.2  |
| .3     | 20+ 60   | 3-5      | -10.7 | 45.0   | -2             | -1.8  | .1              | .2    |
|        |          | 0-0      | -12.1 | 70.0   | -3             | -1.7  | .0              | .3    |
|        |          | -3-5     | -13.4 | -259.6 | -3             | -1.6  | .0              | .3    |
| 1.0    | 20+ 60   | 3-5      | -11.4 | 58.4   | -2             | -7    | .1              | .0    |
|        |          | 0-0      | -12.8 | 86.7   | -2             | -7    | .0              | .1    |
|        |          | -3-5     | -14.1 | -239.5 | -3             | -7    | .0              | .1    |
| 2.0    | 20+ 60   | 3-5      | -12.5 | 80.8   | .0             | -1    | .1              | .0    |
|        |          | 0-0      | -13.8 | -244.1 | -1             | -2    | .0              | -1.1  |
|        |          | -3-5     | -15.0 | -207.3 | -2             | -2    | .0              | -1.2  |

FIG. 3.92

NE 15  
BARRIER HEIGHT= 2-8000+00  
ADMITTANCE= 3-0000+03  
RAVENNUMBER= 2-2800+00

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT |        | BARRIER APPROX |        | RECEIVED APPROX |       |
|--------|----------|----------|-------|--------|----------------|--------|-----------------|-------|
|        |          |          | LEVEL | PHASE  | LEVEL          | PHASE  | LEVEL           | PHASE |
| .3     | 40+ 20   | 3-5      | -22.2 | 67.2   | 3.1            | -10.2  | 5.4             | -2.7  |
|        |          | 0-0      | -23.6 | -196.4 | 1.2            | -5.2   | 1.2             | 2.3   |
|        |          | -3-5     | -26.3 | -7.1   | -2.0           | -6.1   | -1              | 1.4   |
| 1.0    | 40+ 20   | 3-5      | -17.3 | 56.4   | 1.3            | -1     | 2.4             | .9    |
|        |          | 0-0      | -21.1 | -194.0 | -7             | -9     | 1.2             | .0    |
|        |          | -3-5     | -24.4 | 5.3    | -1.5           | -9     | .0              | -1.2  |
| 2.0    | 40+ 20   | 3-5      | -14.9 | 55.9   | .6             | 2.4    | 1.4             | .7    |
|        |          | 0-0      | -16.6 | -181.5 | -5             | 2.1    | .5              | .6    |
|        |          | -3-5     | -23.1 | 31.5   | -7             | 1.5    | .2              | -1.1  |
| .3     | 40+ 40   | 3-5      | -20.4 | 81.8   | .1             | -12.8  | 4.3             | -7.0  |
|        |          | 0-0      | -21.3 | -223.4 | -2             | -6.5   | 1.5             | -7.7  |
|        |          | -3-5     | -25.0 | -130.8 | -3.0           | -5.3   | .2              | .5    |
| 1.0    | 40+ 40   | 3-5      | -17.3 | 72.1   | .0             | -6     | 2.3             | -1.7  |
|        |          | 0-0      | -21.6 | -221.0 | -1.3           | -5     | 1.0             | -6    |
|        |          | -3-5     | -21.6 | -120.4 | -2.0           | .3     | .1              | -0    |
| 2.0    | 40+ 40   | 3-5      | -15.8 | 76.0   | .1             | 2.9    | 1.2             | .4    |
|        |          | 0-0      | -18.6 | -207.2 | -5             | 2.9    | .6              | -4    |
|        |          | -3-5     | -21.0 | -95.5  | -9             | 2.6    | .2              | -8    |
| .3     | 40+ 60   | 3-5      | -19.4 | 86.7   | -1.5           | -11.1  | 3.4             | -8.0  |
|        |          | 0-0      | -20.2 | -234.0 | -3.3           | -5.7   | 1.6             | -8    |
|        |          | -3-5     | -21.4 | -173.7 | -4.8           | -4.0   | .6              | .2    |
| 1.0    | 40+ 60   | 3-5      | -17.2 | 79.8   | .7             | 1.2    | 1.9             | -3.0  |
|        |          | 0-0      | -18.6 | -231.6 | -1.7           | -1.4   | 1.0             | -1.4  |
|        |          | -3-5     | -20.3 | -164.2 | -2.3           | 1.6    | .4              | -1.6  |
| 2.0    | 40+ 60   | 3-5      | -16.3 | 86.6   | -2             | -356.0 | .9              | -7    |
|        |          | 0-0      | -18.3 | -216.2 | -6             | 4.1    | .6              | -6    |
|        |          | -3-5     | -20.1 | -140.2 | -9             | 3.9    | .2              | -8    |

FIG. 3.94

NE 15  
BARRIER HEIGHT= 2-8000+00  
ADMITTANCE= 3-0000+03  
RAVENNUMBER= 2-2800+00

THE DIFFRACTED REFLECTED FIELD

| SOURCE | DISTANCE | RECEIVER | EXACT |        | BARRIER APPROX |       | RECEIVED APPROX |       |
|--------|----------|----------|-------|--------|----------------|-------|-----------------|-------|
|        |          |          | LEVEL | PHASE  | LEVEL          | PHASE | LEVEL           | PHASE |
| .3     | 40+ 20   | 3-5      | -9.3  | 12.6   | .5             | -1.6  | .4              | .2    |
|        |          | 0-0      | -13.2 | 71.9   | .0             | -2.1  | -1              | .3    |
|        |          | -3-5     | -16.6 | -198.6 | -2             | -2.6  | .0              | -1.4  |
| 1.0    | 40+ 20   | 3-5      | -9.4  | 17.4   | .3             | -1.3  | .3              | .2    |
|        |          | 0-0      | -13.4 | 80.5   | .0             | -1.6  | .1              | .0    |
|        |          | -3-5     | -16.7 | -175.4 | -2             | -2.0  | .0              | -1.4  |
| 2.0    | 40+ 20   | 3-5      | -9.6  | 24.1   | .1             | -1    | .2              | .4    |
|        |          | 0-0      | -13.6 | -267.1 | -1             | -1.0  | .0              | -1.1  |
|        |          | -3-5     | -16.8 | -156.3 | -2             | -1.3  | .0              | -1.1  |
| .3     | 40+ 40   | 3-5      | -10.0 | 21.8   | .9             | -3.2  | .2              | .0    |
|        |          | 0-0      | -12.3 | 57.3   | .0             | -3.2  | .0              | .0    |
|        |          | -3-5     | -14.6 | -251.7 | -2             | -3.4  | .0              | -1.1  |
| 1.0    | 40+ 40   | 3-5      | -10.1 | 27.9   | .2             | -2.4  | .2              | .3    |
|        |          | 0-0      | -12.6 | 66.4   | .1             | -2.4  | .1              | .3    |
|        |          | -3-5     | -14.8 | -239.3 | -2             | -2.5  | .0              | -1.2  |
| 2.0    | 40+ 40   | 3-5      | -10.5 | 36.9   | .0             | -1.4  | .2              | .3    |
|        |          | 0-0      | -12.9 | 78.9   | .0             | -1.4  | .0              | .3    |
|        |          | -3-5     | -15.1 | -221.0 | -2             | -1.5  | .0              | -1.2  |
| .3     | 40+ 60   | 3-5      | -10.3 | 26.0   | .3             | -4.2  | .2              | .1    |
|        |          | 0-0      | -12.0 | 52.6   | .0             | -4.0  | .0              | .0    |
|        |          | -3-5     | -13.6 | 86.7   | -2             | -4.0  | .0              | .0    |
| 1.0    | 40+ 60   | 3-5      | -10.5 | 33.6   | .1             | -3.0  | .1              | .2    |
|        |          | 0-0      | -12.2 | 61.8   | .0             | -2.9  | .0              | .1    |
|        |          | -3-5     | -13.9 | -261.5 | -3             | -2.9  | .0              | .1    |
| 2.0    | 40+ 60   | 3-5      | -10.9 | 43.9   | .0             | -1.0  | .1              | .1    |
|        |          | 0-0      | -12.6 | 82.8   | .0             | -1.0  | .0              | .0    |
|        |          | -3-5     | -14.3 | -243.7 | -3             | -1.0  | .0              | .1    |





### 3.5. The reduction of sound by a wall on level grassland

The measurements have all been made on measurement site 1 with the 2.8 m high wall. As before the reference sound pressure level is the free field level + 6 dB. The notation  $D = x + y$  indicates that the distance source-wall and wall-receiver is  $x$  m and  $y$  m respectively.

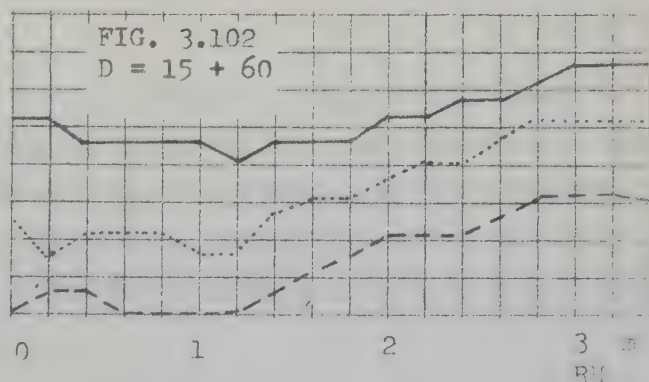
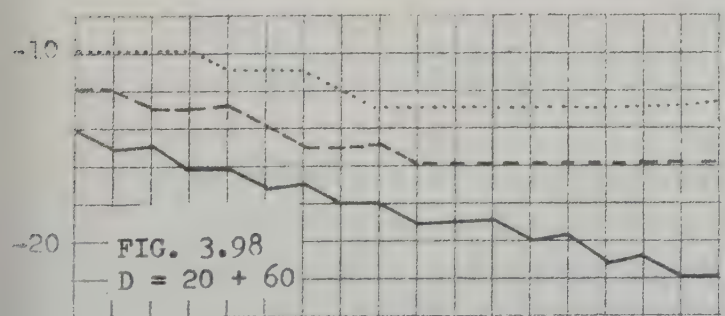
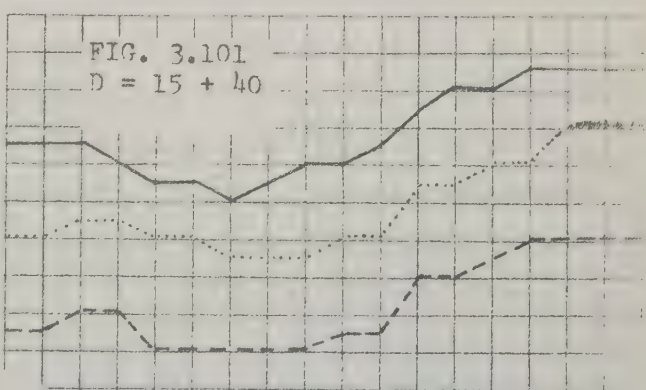
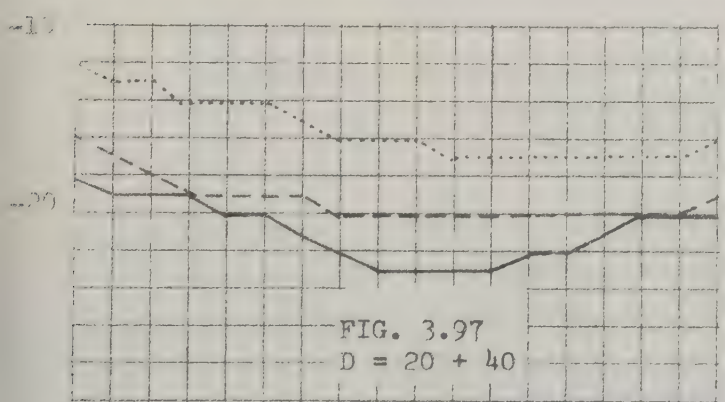
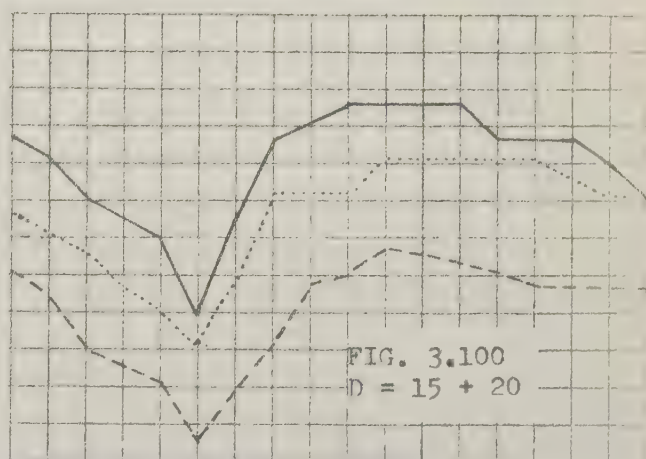
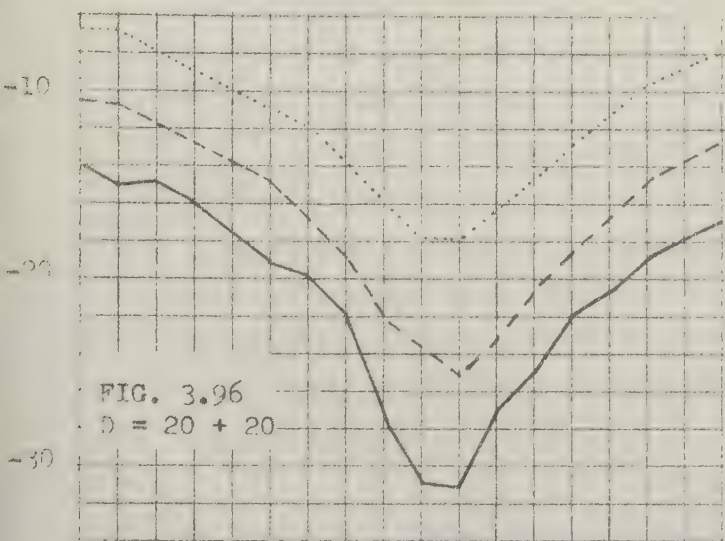
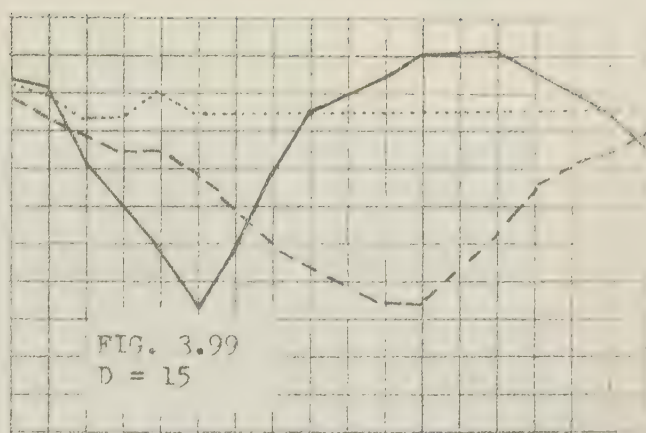
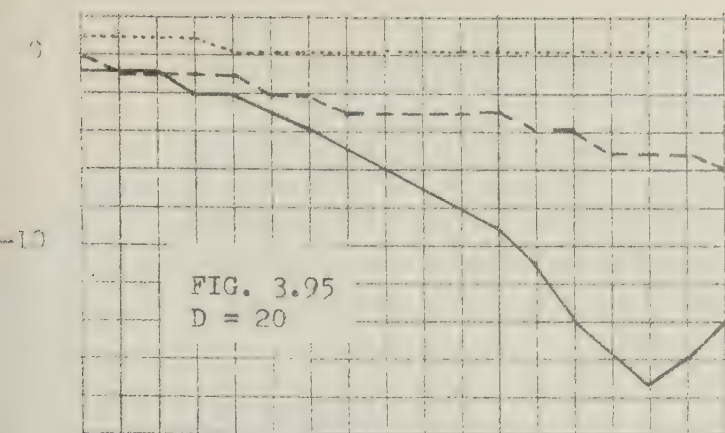
#### 3.5.1. SOME PHYSICALLY INTERESTING MEASUREMENTS

In Figs. 3.95 - 3.102 some specially selected measurements have been coupled together to illustrate the great importance of all parameters on both sides of the barrier.

Figs. 3.96 - 3.98 and 3.100 - 3.102 indicate that the height dependence of the sound pressure level on the receiver's side of the barrier is almost independent of the position of the source. It is also obvious that the general level of the curves to a great extent is determined by the sound pressure level at the top of the barrier. The relative order between the curves is always the same as the one which is found at 2.8 m in Figs. 3.95 and 3.99.

The above behaviour follows directly from the reasoning in Section 2.3.2.4 and especially Eq. (2.111) providing that the ground impedance is infinite. For finite impedances something similar will happen as long as the barrier approximation of Section 3.4 does not differ too much from the receiver approximation. In these cases it is possible to get a qualitative estimation of the effect of an acoustic barrier by separating the total attenuation into 3 different ones, namely the ground attenuation between source and barrier, the reduction due to diffraction and the ground attenuation between barrier and receiver.





FIGS. 3.95 - 3.98. 250 Hz

— SH = 2.0 — — SH = 1.0 . . . SH = 0.3

FIGS. 3.99 - 3.102. 500 Hz

— SH = 2.0 — — SH = 1.0 . . . SH = 0.3





### 3.5.2. COMPARISONS BETWEEN CALCULATED AND MEASURED VALUES

Eq. (2.139) has been used for all computations except for those marked "exact" when Eq. (2.135) was used. The different values of the ground admittance are identical to those in Section 3.2.1. The results are accounted for in Figs. 3.102 - 3.174.

Some comments:

The agreement between the "exact" and the approximate solution in those cases which have been dealt with is good. A maximum deviation of about 2 dB is found in Figs. 3.136 and 3.138, that is at 250 Hz when  $\underline{v} = 0.03 - i \cdot 0.02$ . The fact of this being the worst case could have been anticipated from the study of the diffracted reflected field in Fig. 3.92. Bearing the potential influence of weather conditions and other uncertain factors in mind deviations of this magnitude should be acceptable. It must however be emphasized that under certain extreme conditions, which are evident from the theoretical analysis in Sections 2.3 and 2.4, the approximate method is likely to fail.

At 125 Hz an infinite ground impedance generally yields acceptable results. There are however some exceptions which seem to have been caused by erratic measurements. A consequence of the reciprocity theorem is that the measured curve in Fig. 3.115, which must be lowered about 2 dB due to the increased power output close to the ground, is inconsistent with those in Figs. 3.116 and 3.117. Assuming the measured curve in Fig. 3.115 is correct this means that the other two experimental curves must be lowered about 3 dB. This result is more reasonable.

At 250 Hz the best agreement between calculated and measured values is given with  $\underline{v} = 0.03 - i \cdot 0.02$ . This admittance also yielded the best results on the same measurement site but without the wall.

At 500 Hz  $\underline{v} = 0.06 - i \cdot 0.06$  gives better agreement with the experimental values than  $\underline{v} = 0.03 - i \cdot 0.02$ , while the opposite was the case without the wall. In that case the difference between the two admittances was, however, small for large values of SH and RH. Under these conditions  $\underline{v} = 0.06 - i \cdot 0.06$  even yielded the best results at 40 m and 80 m. Accordingly it is not surprising

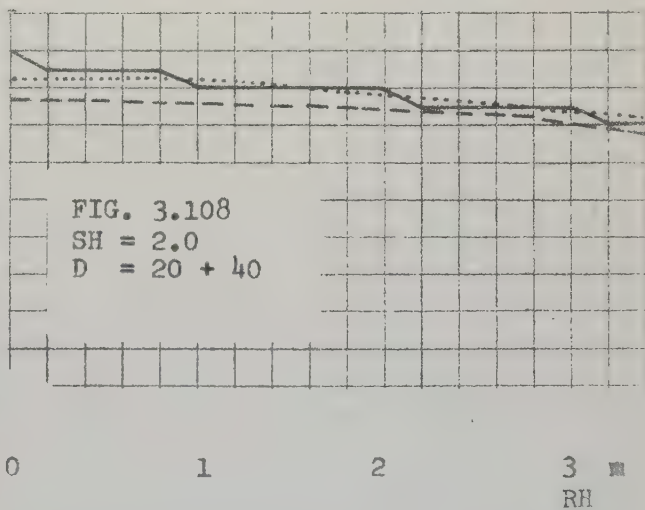
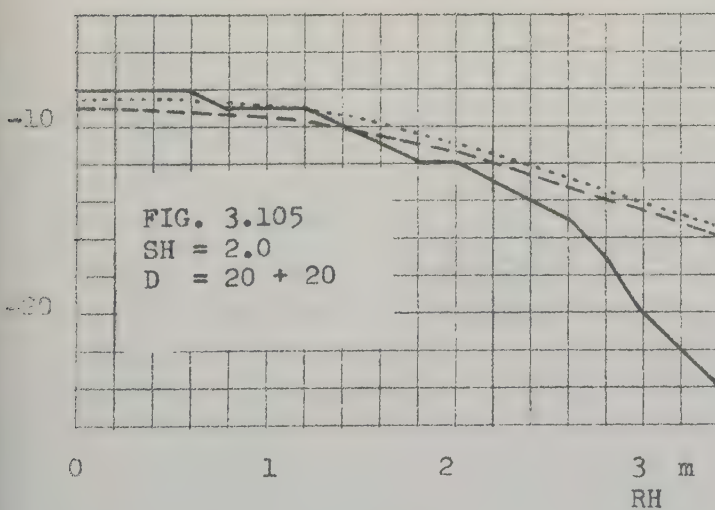
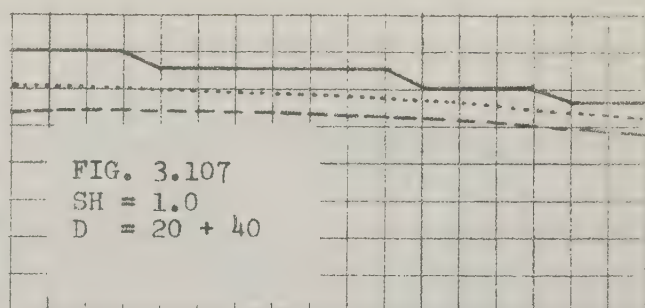
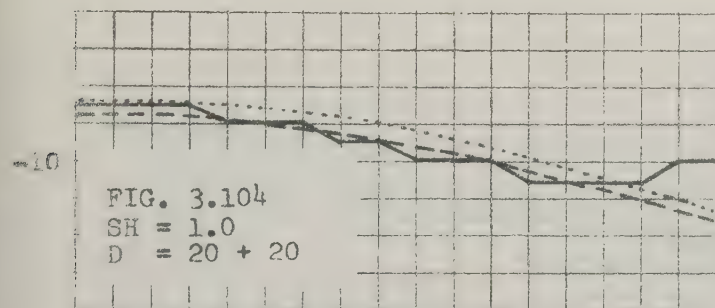
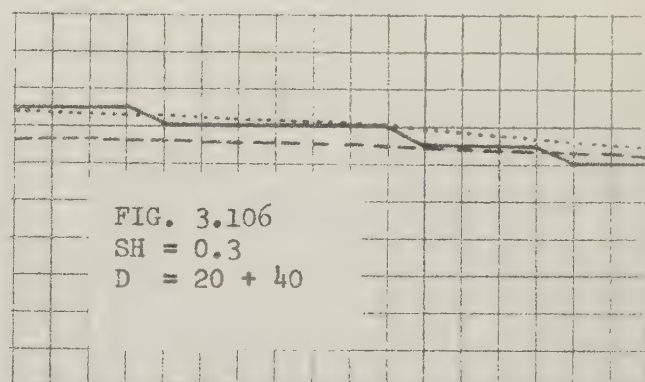
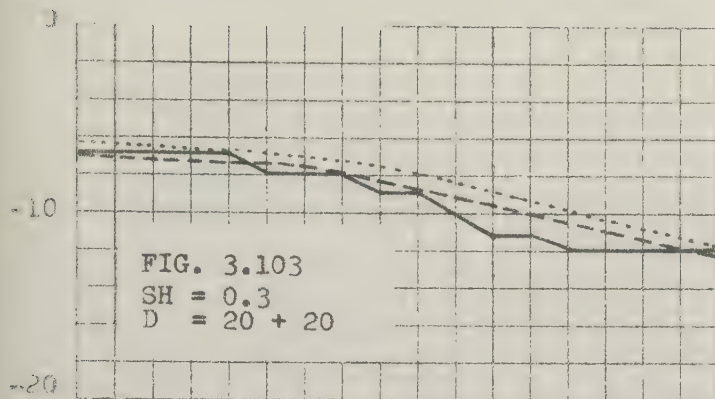




that this admittance was the most suitable one to use when carrying out computations from the same measurement site with a high wall.

At 1000 Hz  $\underline{v} = 0.10 - i \cdot 0.05$  gives a slightly better agreement with the measured values than  $\underline{v} = 0.05 - i \cdot 0.02$ . This was also the case without the wall.

dB



FIGS. 3.103 - 3.108

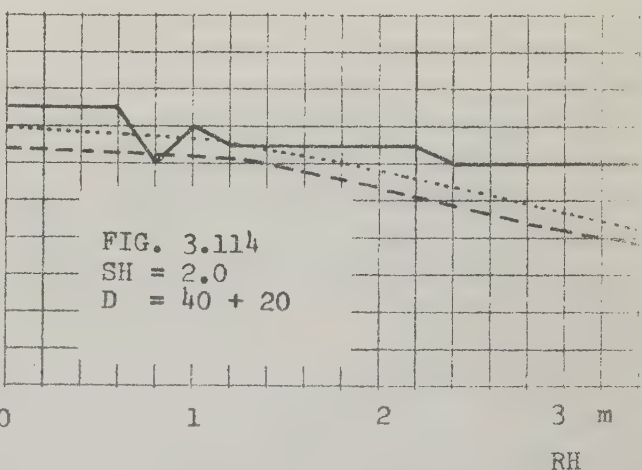
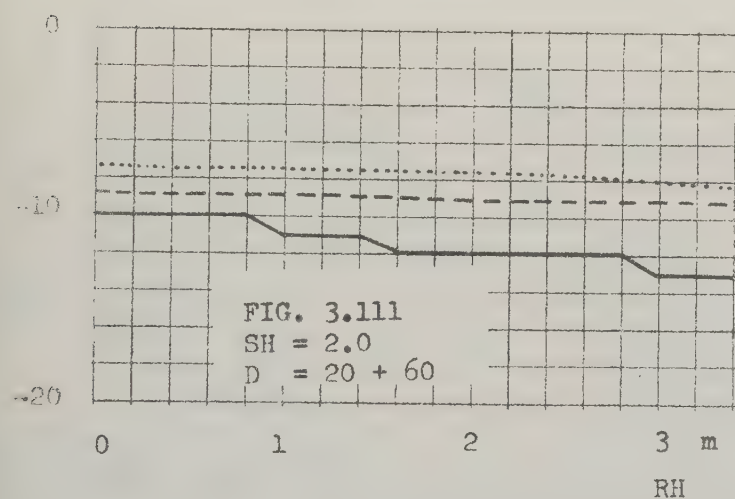
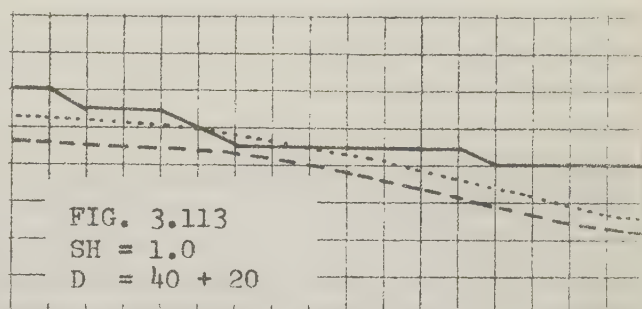
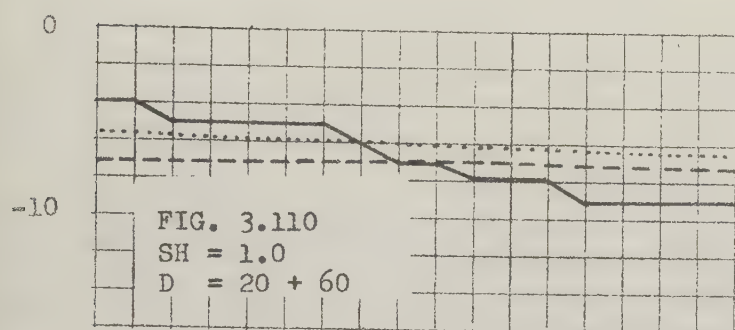
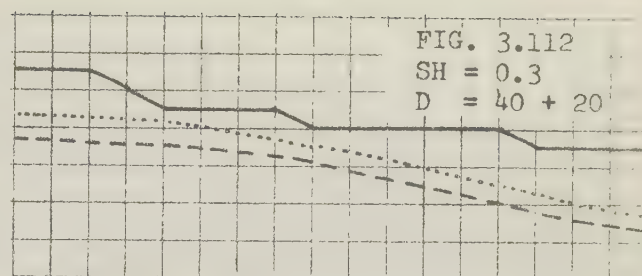
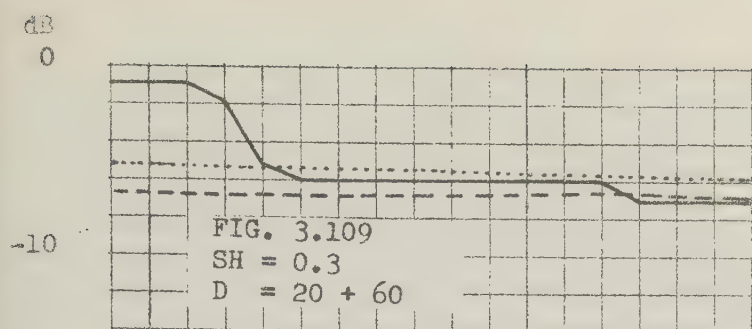
125 Hz

— measured

- - -  $\underline{v} = 0.005 - i \cdot 0.005$

. . .  $\underline{v} = 0.000 - i \cdot 0.000$





FIGS. 3.109 - 3.114

125 Hz

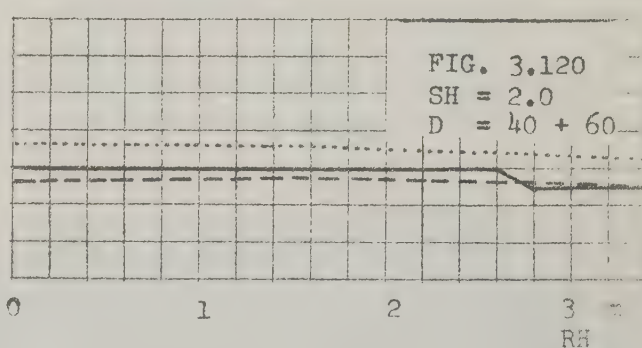
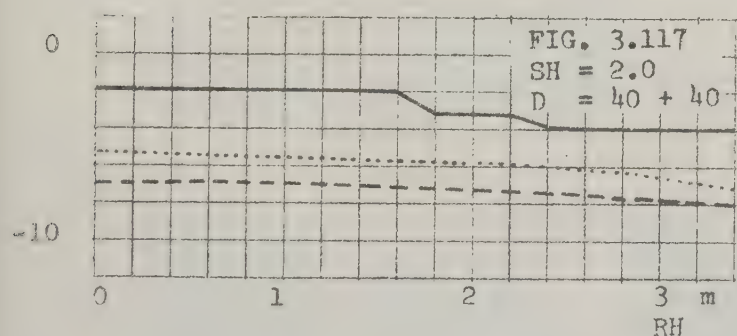
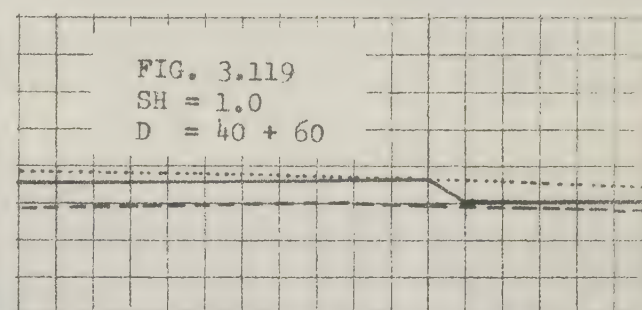
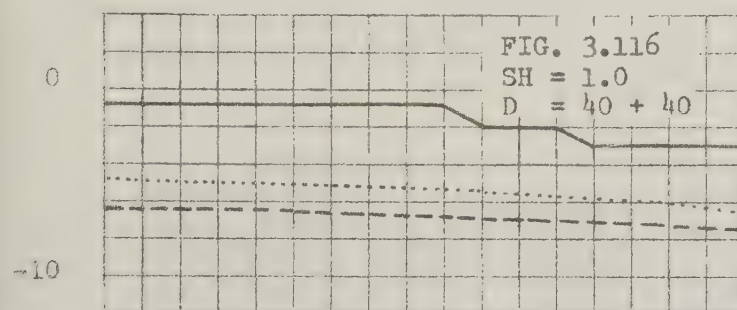
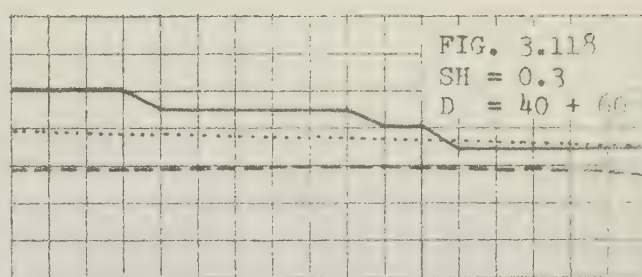
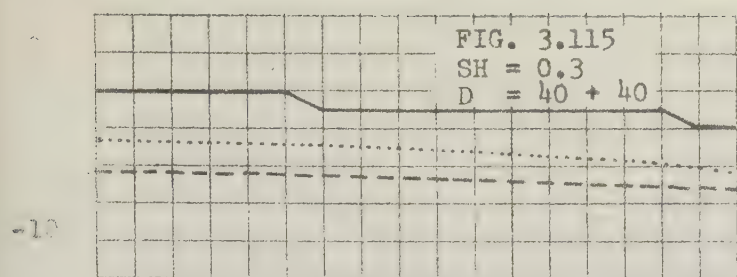
— measured

- - -  $\underline{v} = 0.005 - i \cdot 0.005$ . . .  $\underline{v} = 0.000 - i \cdot 0.000$





dB



FIGS. 3.115 - 3.120

125 Hz

— measured

- - -  $\underline{u} = 0.005 - i \cdot 0.005$ . . .  $\underline{u} = 0.000 - i \cdot 0.000$



dB  
0

-10

-20

FIG. 3.121  
SH = 0.3  
D = 20 + 20

FIG. 3.124  
SH = 0.3  
D = 20 + 40

-10

-20

FIG. 3.122  
SH = 1.0  
D = 20 + 20

FIG. 3.125  
SH = 1.0  
D = 20 + 40

-10

-20

-30

FIG. 3.123  
SH = 2.0  
D = 20 + 20

FIG. 3.126  
SH = 2.0  
D = 20 + 40

0 1 2 3 m  
RH

0 1 2 3 m  
RH

FIGS. 3.121 - 3.126

250 Hz

— measured

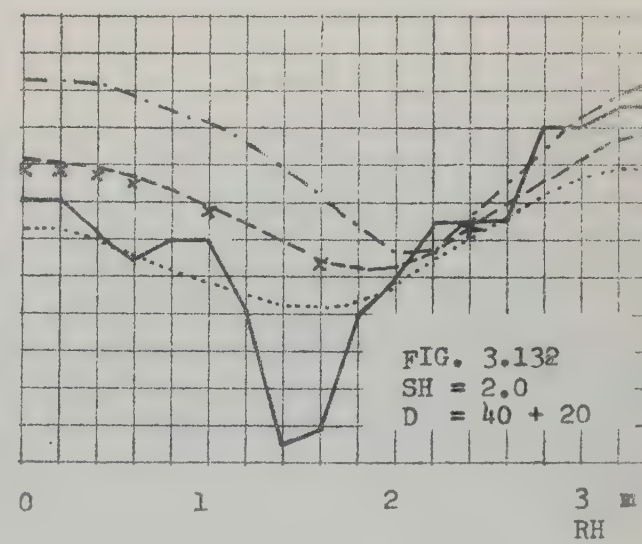
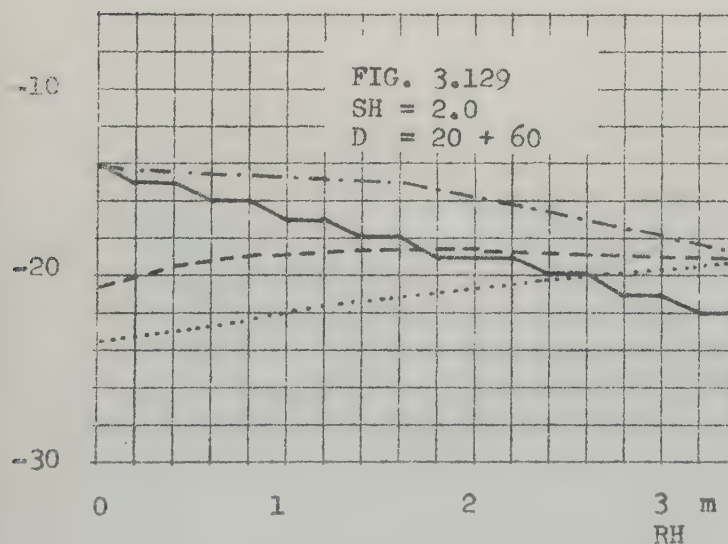
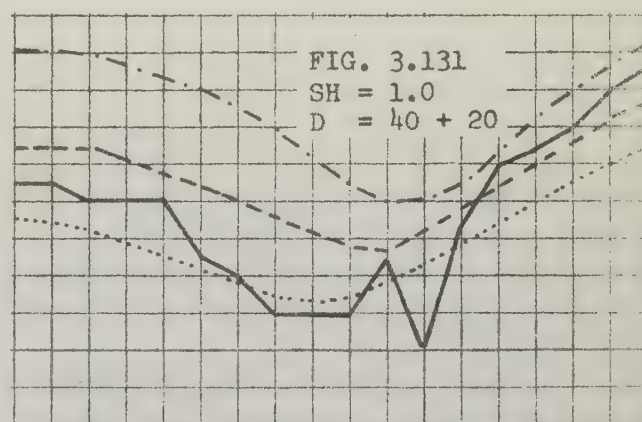
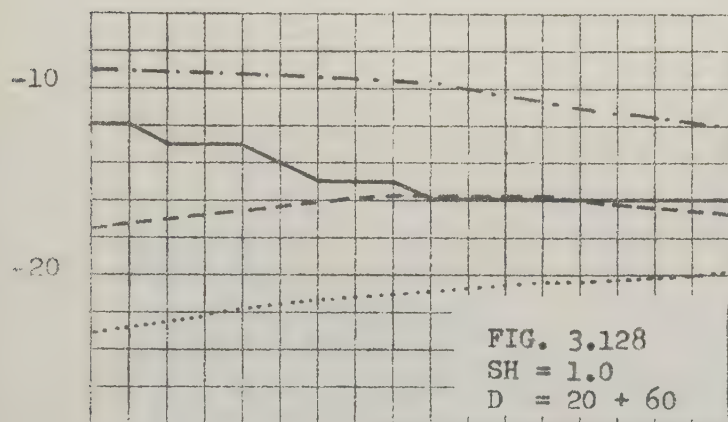
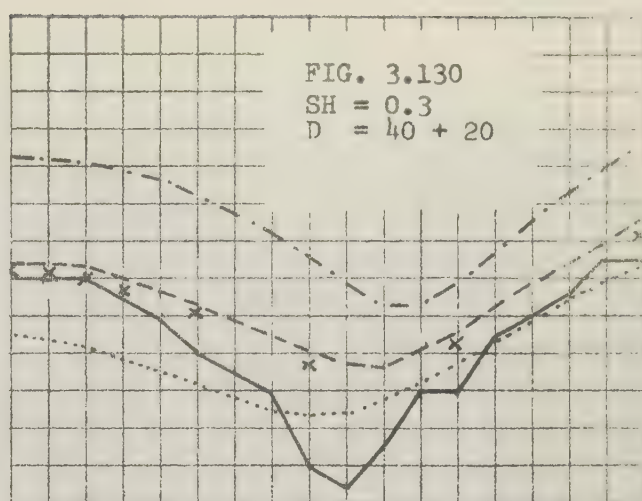
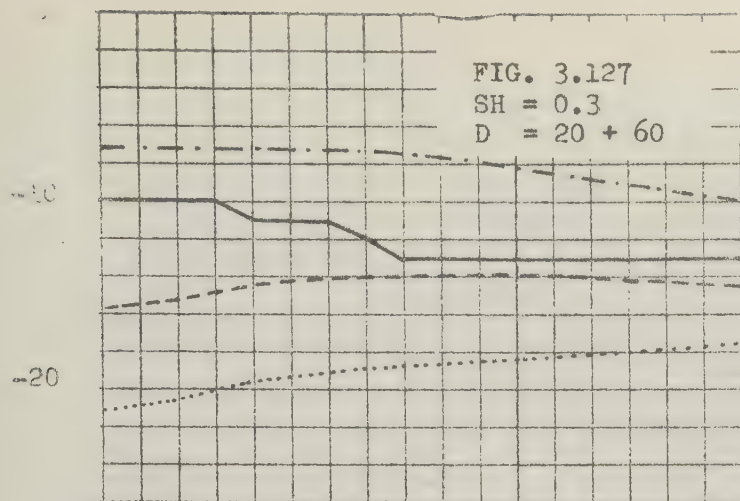
- - -  $\underline{u} = 0.03 - i \cdot 0.02$

. . .  $\underline{u} = 0.05 - i \cdot 0.05$

- . -  $\underline{u} = 0.00 - i \cdot 0.00$







FIGS. 3.127 - 3.132

250 Hz

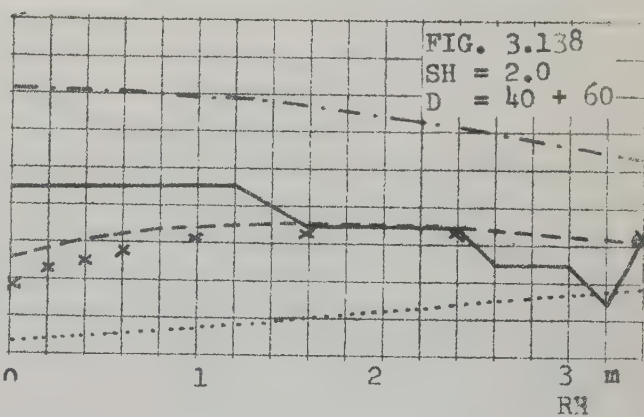
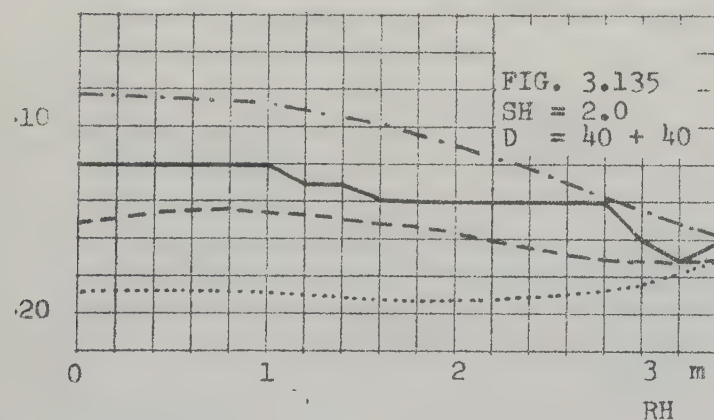
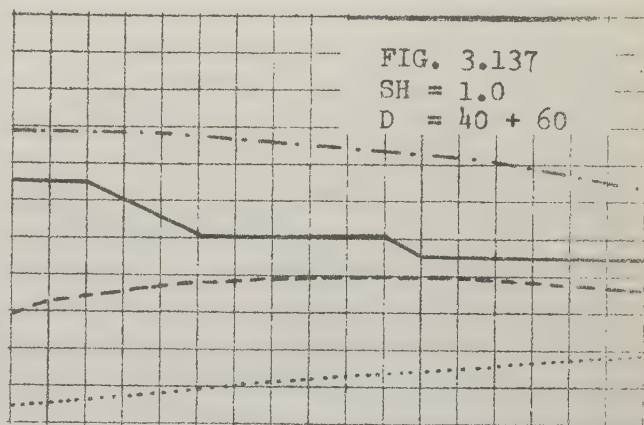
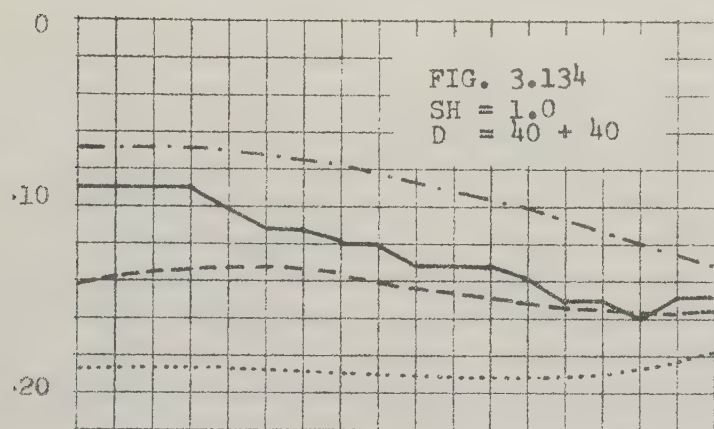
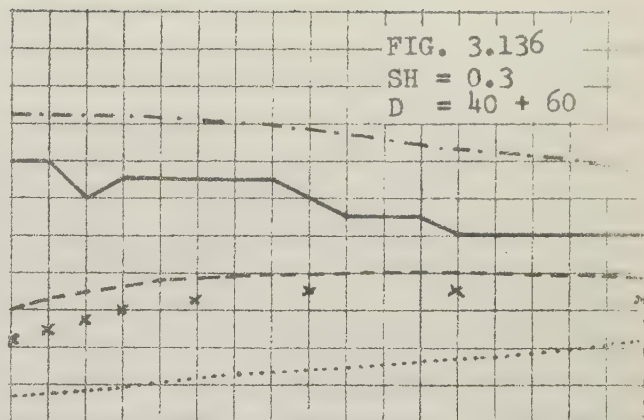
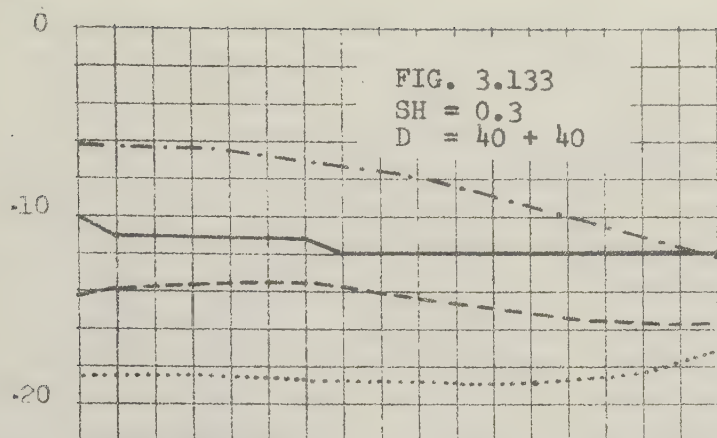
— measured

- - -  $\underline{u} = 0.03 - i \cdot 0.02$  X = "exact". . .  $\underline{u} = 0.05 - i \cdot 0.05$ - . -  $\underline{u} = 0.00 - i \cdot 0.00$





dB



FIGS. 3.133 - 3.138

250 Hz

— measured

- - -  $\underline{u} = 0.03 - i \cdot 0.02$  X = "exact". . .  $\underline{u} = 0.05 - i \cdot 0.05$ - . -  $\underline{u} = 0.00 - i \cdot 0.00$

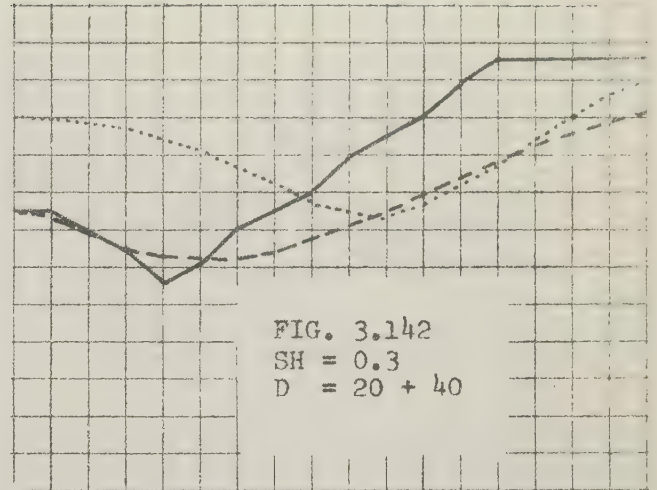
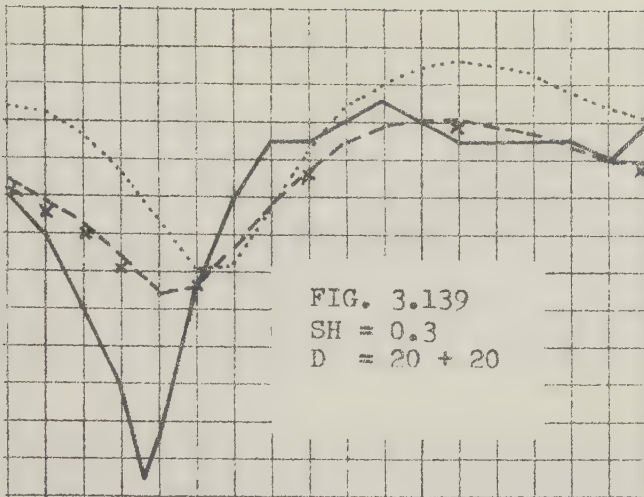


B

10

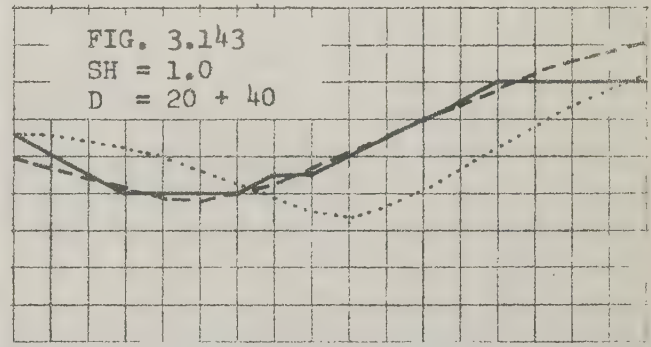
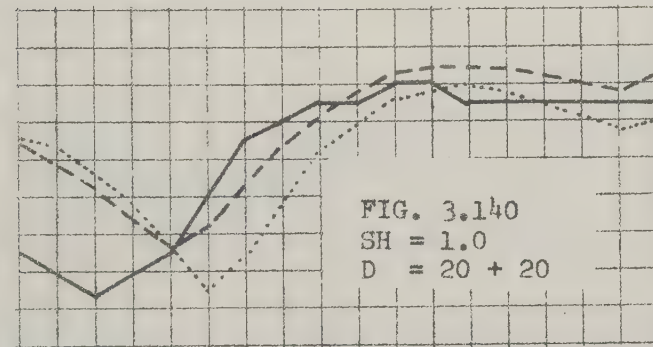
20

30



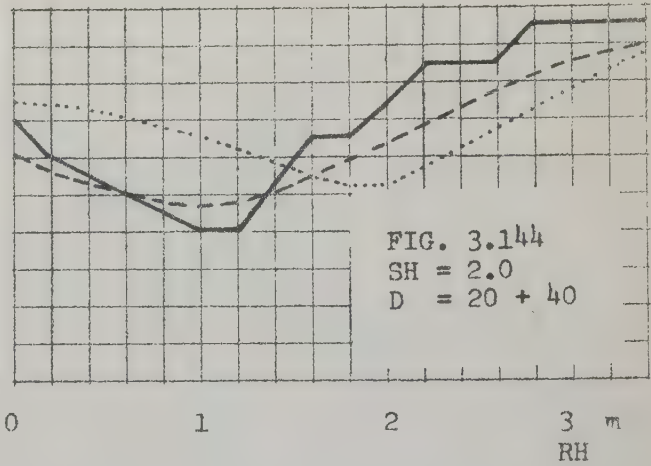
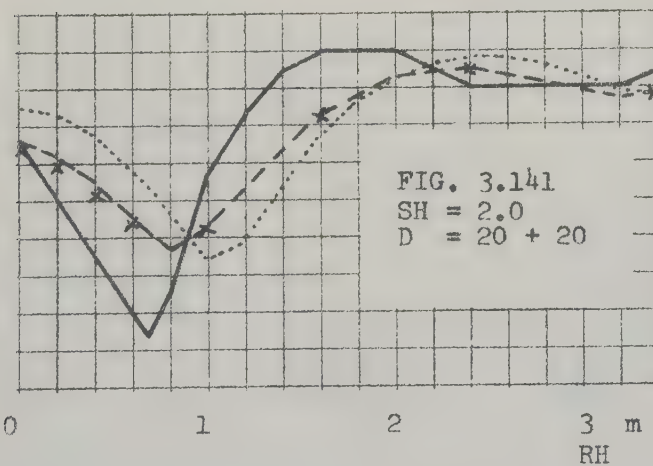
-20

-30



-10

-20



FIGS. 3.139 - 3.144

500 Hz

— measured

- - -  $\underline{u} = 0.06 - i \cdot 0.06$  X = "exact"...  $\underline{u} = 0.03 - i \cdot 0.02$





dB

-10

-20

-30

FIG. 3.145  
SH = 0.3  
D = 20 + 60

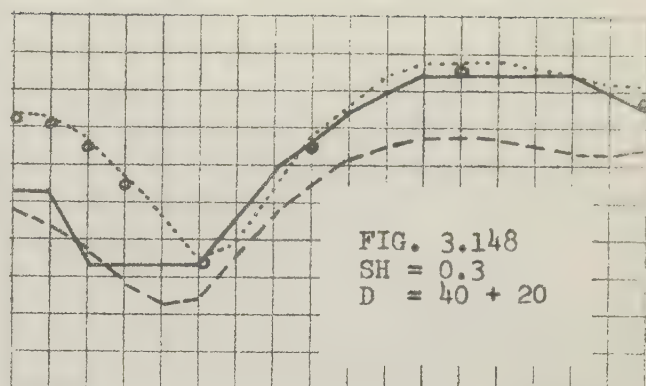


FIG. 3.148  
SH = 0.3  
D = 40 + 20

-10

-20

-30

FIG. 3.146  
SH = 1.0  
D = 20 + 60

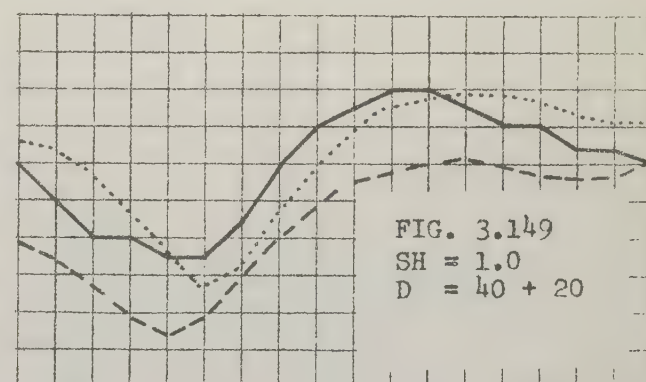


FIG. 3.149  
SH = 1.0  
D = 40 + 20

-10

-20

-30

FIG. 3.147  
SH = 2.0  
D = 20 + 60

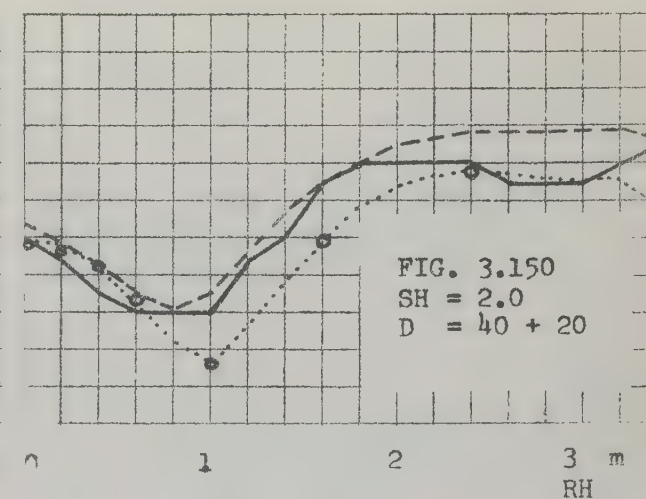


FIG. 3.150  
SH = 2.0  
D = 40 + 20

0 1 2 3 m  
RH

FIGS. 3.145 - 3.150

500 Hz

— measured

- - -  $\underline{u} = 0.06 - i \cdot 0.06$

. . .  $\underline{u} = 0.03 - i \cdot 0.02$       o = "exact"

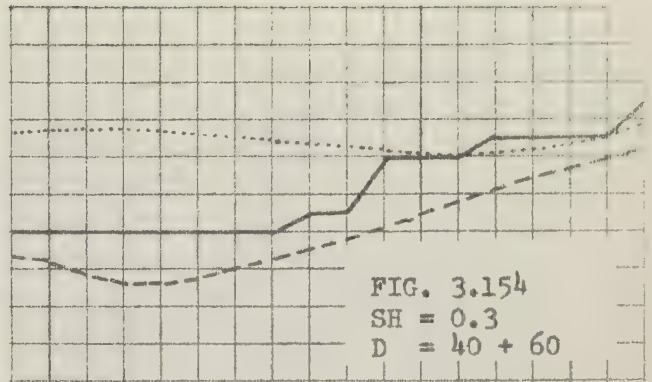
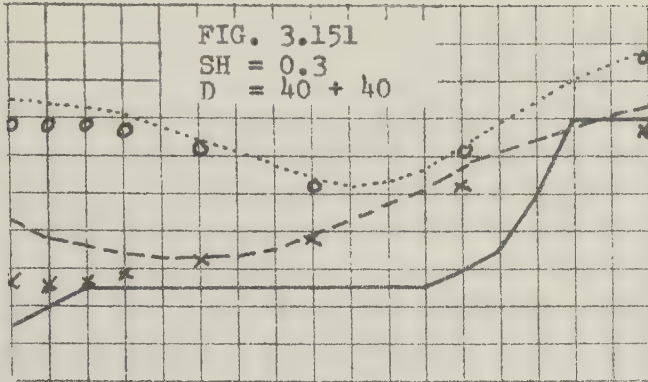


dB

-10

-20

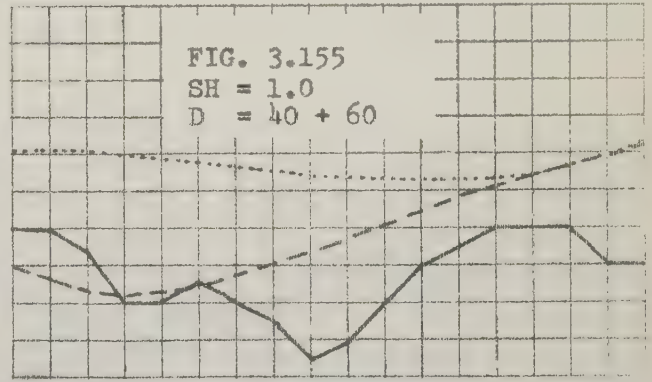
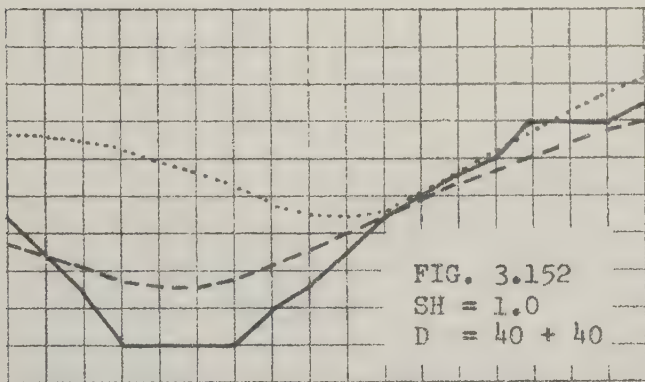
-30



-10

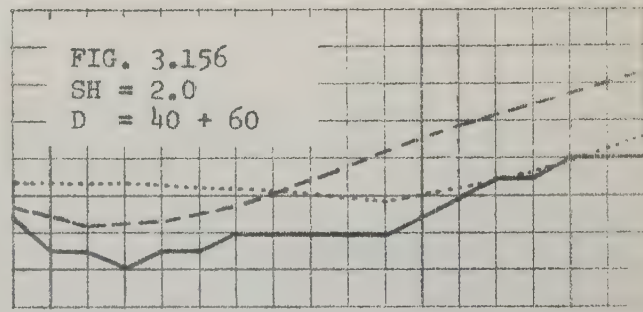
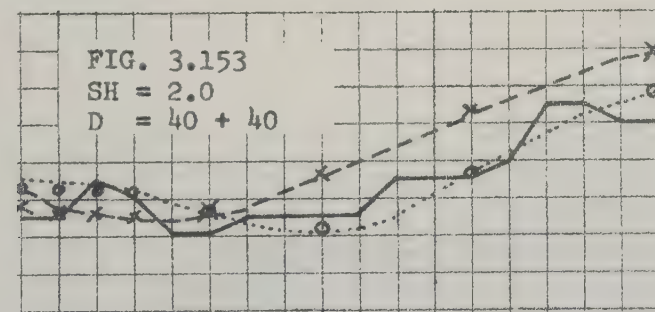
-20

-30



-10

-20



0 1 2 3 m 0 1 2 3 m  
RH RH

FIGS. 3.151 - 3.156

500 Hz

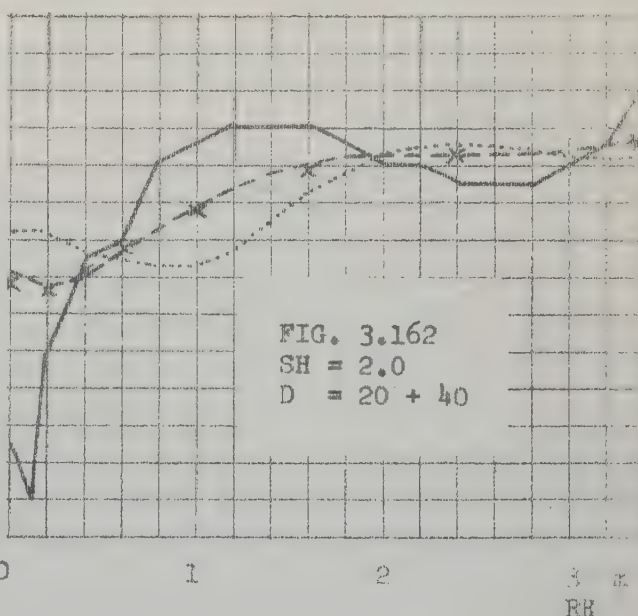
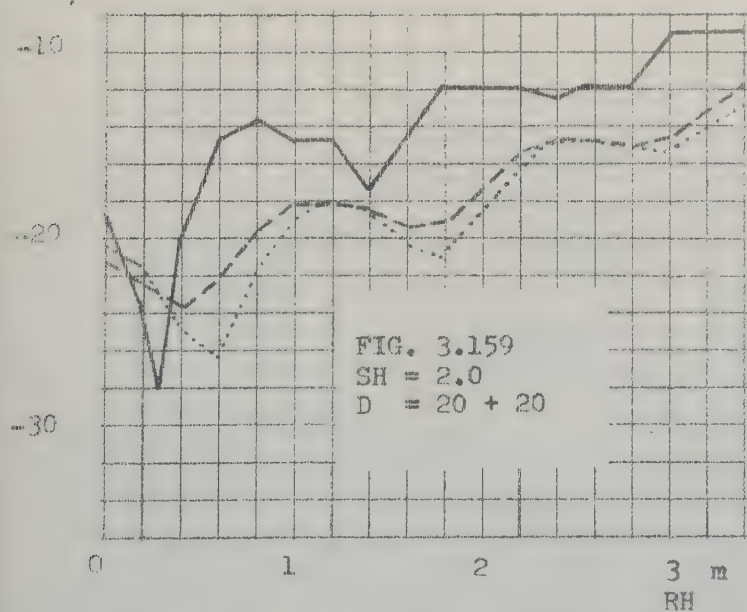
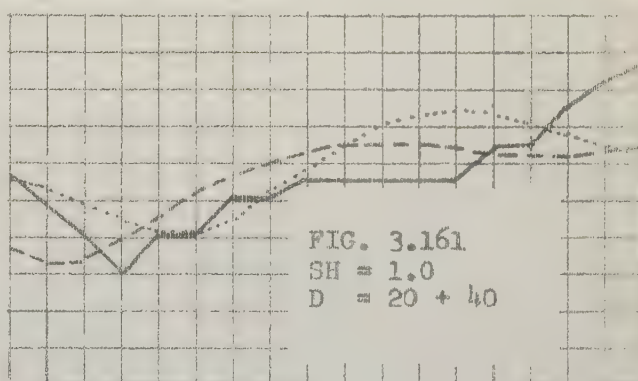
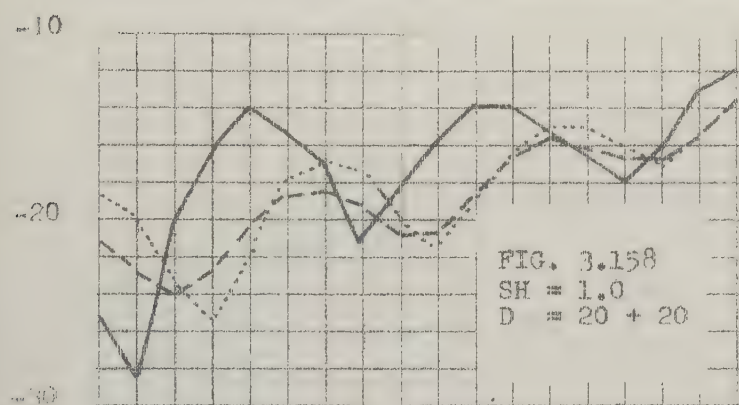
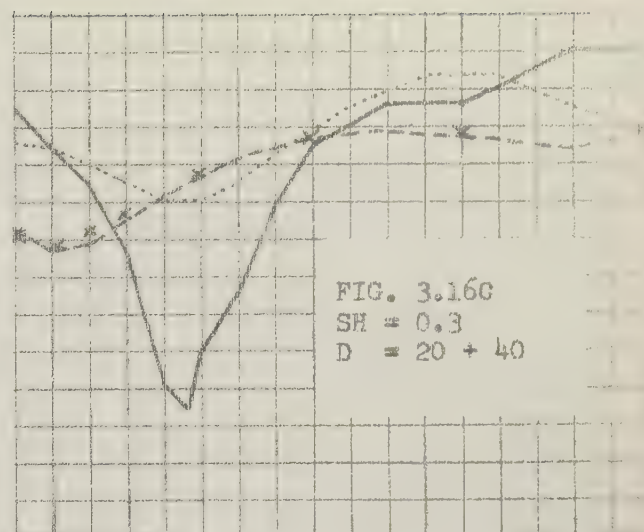
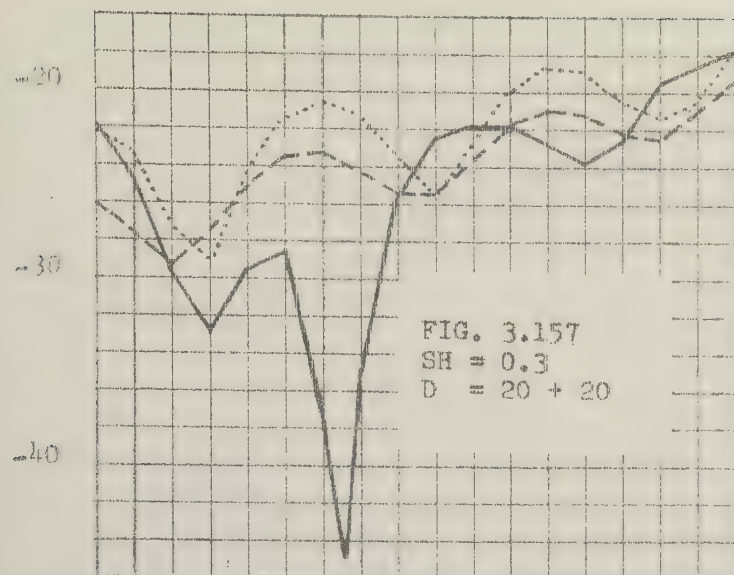
— measured

- - -  $\underline{u} = 0.06 - i \cdot 0.06$  x = "exact"

. . .  $\underline{u} = 0.03 - i \cdot 0.02$  o = "exact"







FIGS. 3.157 - 3.162

1000 Hz

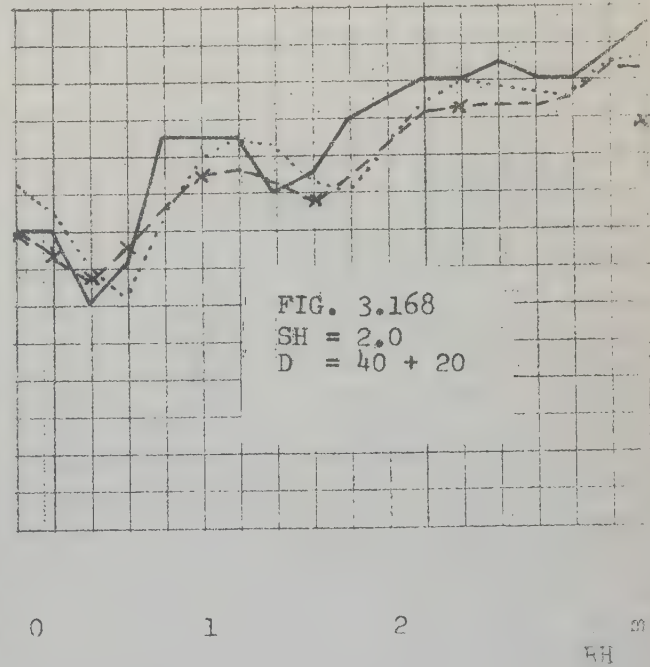
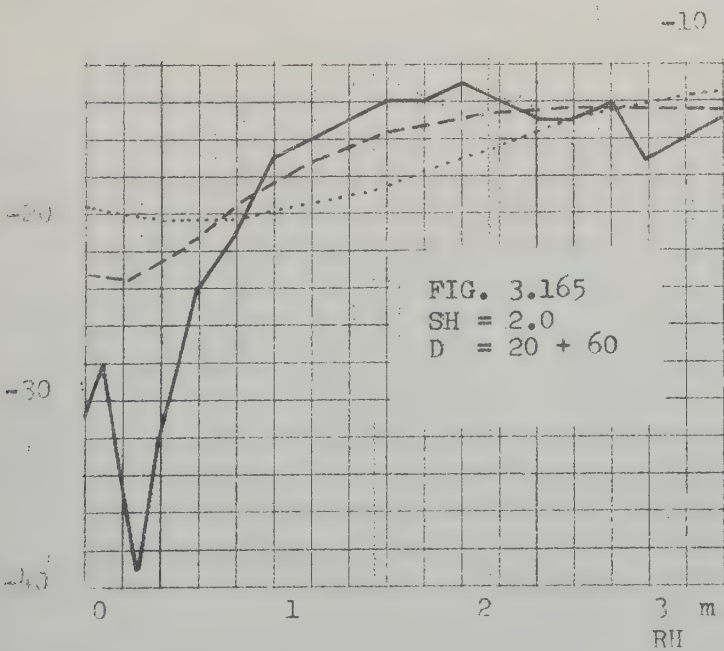
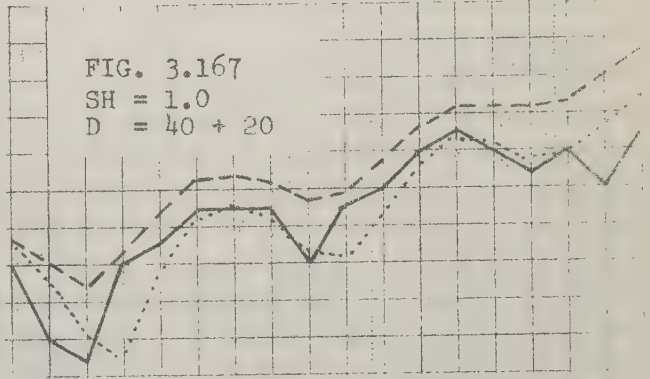
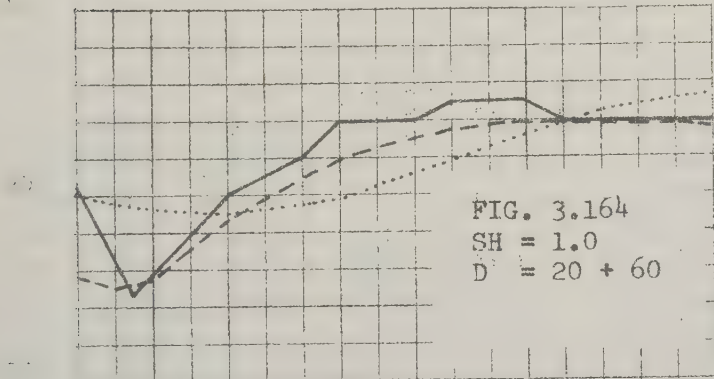
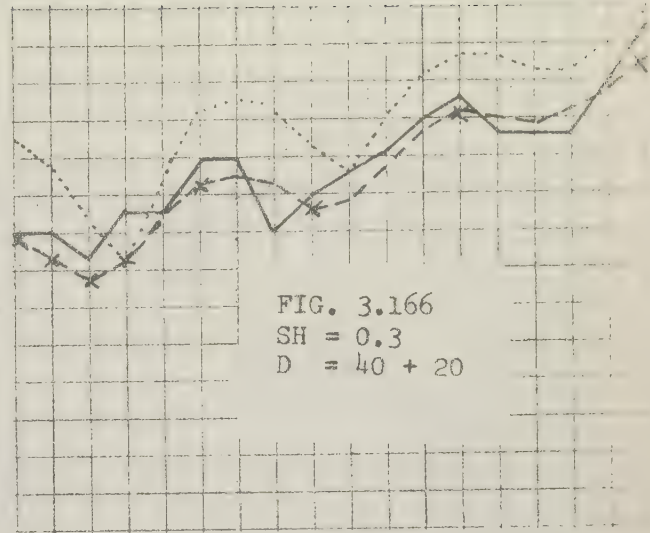
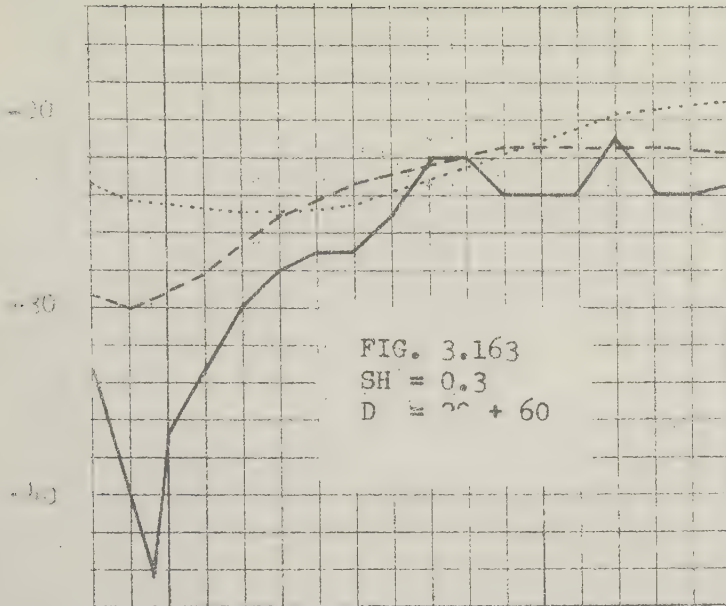
— measured

---  $\nu = 0.10 - i \cdot 0.05$  X = "exact"...  $\nu = 0.05 - i \cdot 0.02$





dB

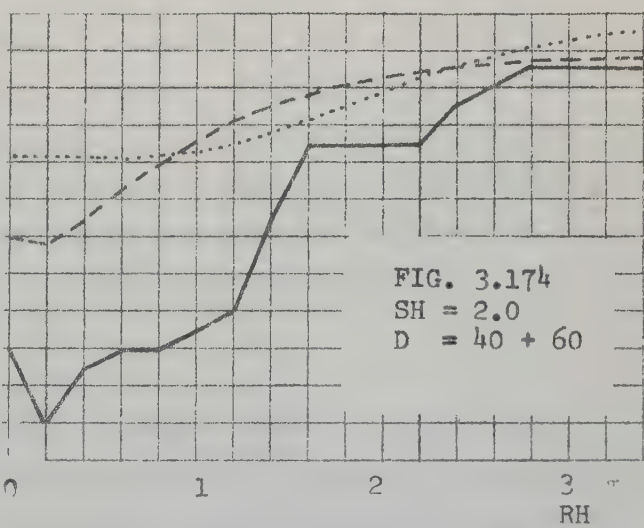
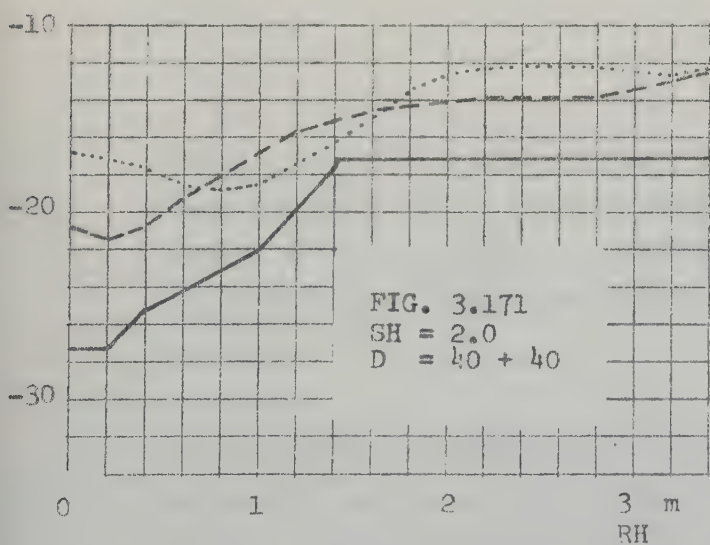
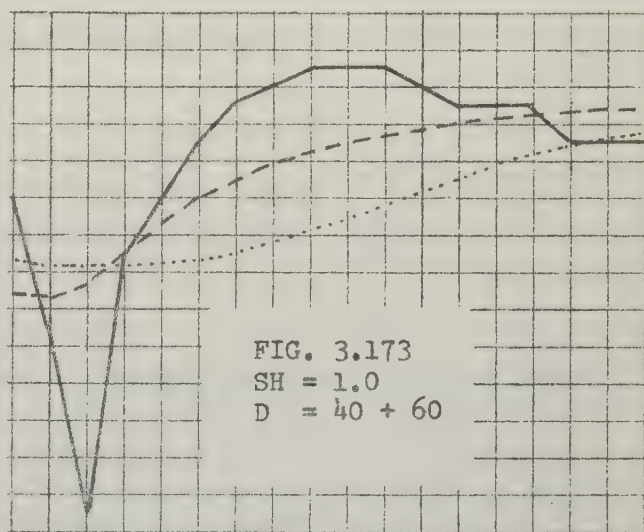
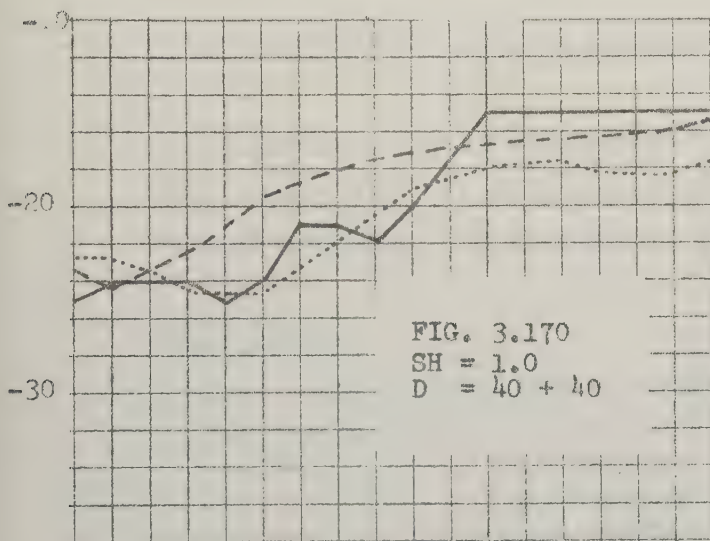
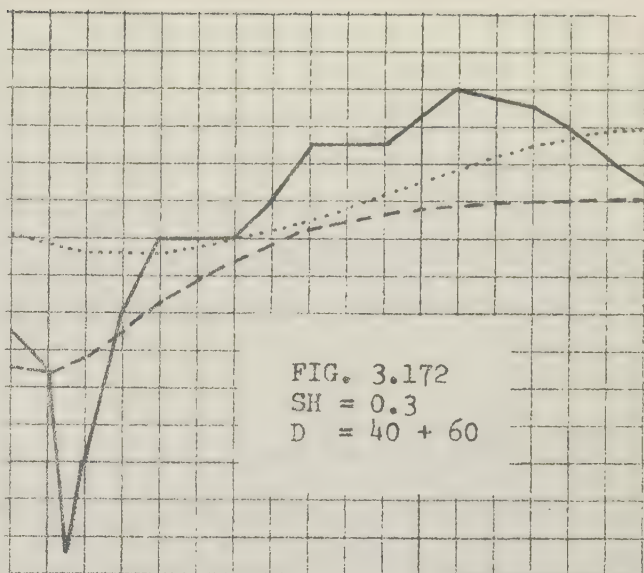
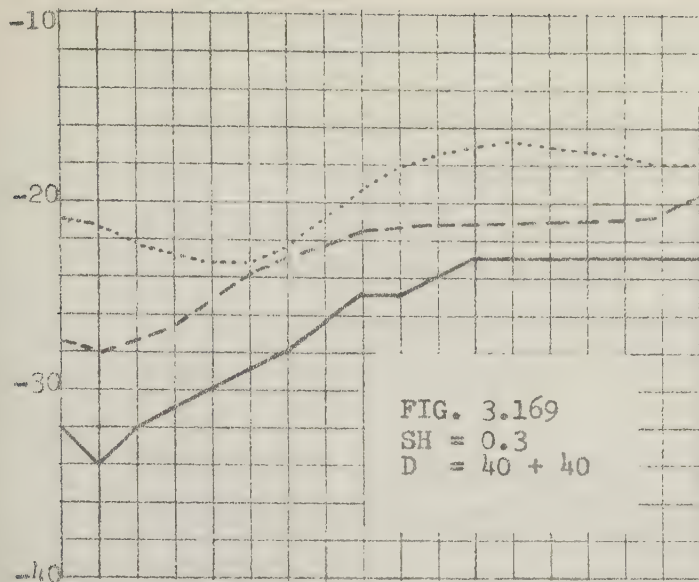


FIGS. 3.163 - 3.168 1000 Hz

— measured      .....  $\underline{u} = 0.05 - i \cdot 0.02$   
 - - -  $\underline{u}' = 0.10 - i' \cdot 0.05$       X = "exact"



dB



FIGS. 3.169 - 3.174

1000 Hz

— measured

- - -  $u = 0.10 - i \cdot 0.05$

. . .  $u = 0.05 - i \cdot 0.02$





### 3.6. The insertion loss of an acoustic barrier

The insertion loss of an acoustic barrier is defined as the difference between the sound pressure level at one point in space before and after the barrier has been inserted between this point and the sound source. As soon as we want to decrease a known noise level at a particular place by erecting a screen of some kind we are interested in the insertion loss and not in the total sound reduction.

The computed examples in Sections 3.6.2. and 3.6.3 have been included because they may have some practical applications. Further details are given in Section 4.

#### 3.6.1. COMPARISONS BETWEEN CALCULATED AND MEASURED VALUES ON LEVEL GRASSLAND

All computations have been carried out in the same way as those in Sections 3.2.1 and 3.5.2. The measurement points have been exactly the same with and without the wall. The results are accounted for in Figs. 3.175 - 3.198.

Some comments:

In Figs. 3.178 and 3.180 the experimental curves have been excluded because of too high background levels.

In a free field high frequencies are always reduced more by a barrier than low ones. When the ground is present this must no longer be the case. A striking result of this section is that the insertion loss often is less at high frequencies than at 125 Hz. This depends on the fact that a high barrier generally significantly decreases the ground attenuation on both sides of the barrier. This is easy to realize by studying the propagation curves in Section 3.2.1 for different source and receiver heights. It is also self-evident that the insertion loss does not necessarily have to be positive.

If the ground impedance is infinite the agreement between measured and calculated values is non-existent at 500 and 1000 Hz, both bad and good at 250 Hz and good at 125 Hz. This indicates that the finite ground impedance must be taken into account at all but the very lowest frequencies.

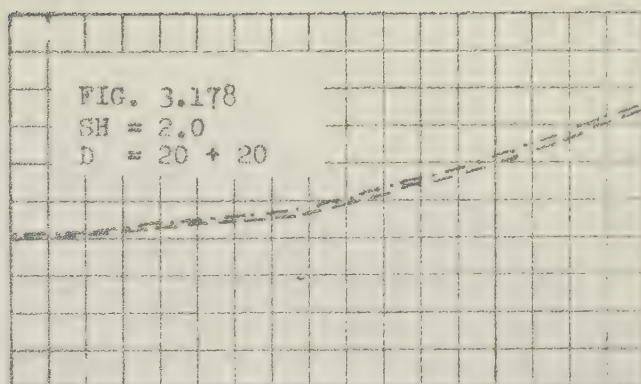
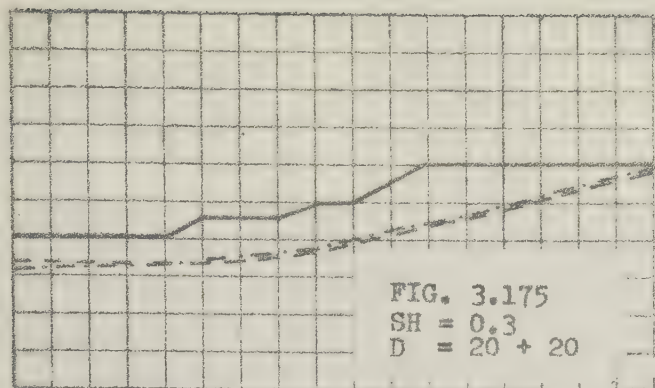


dB

20

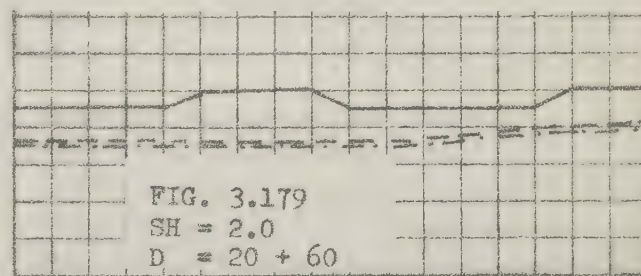
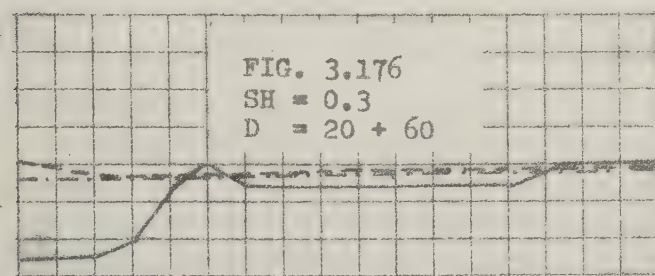
10

0



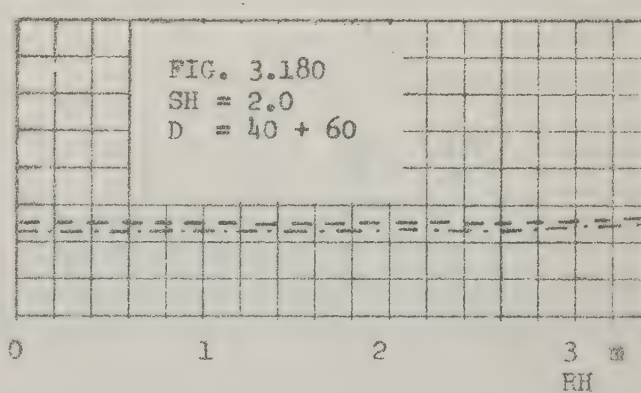
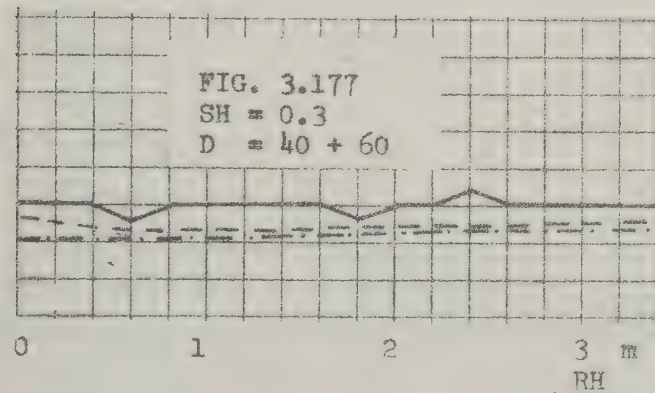
10

0



10

0



FIGS. 3.175 - 3.180

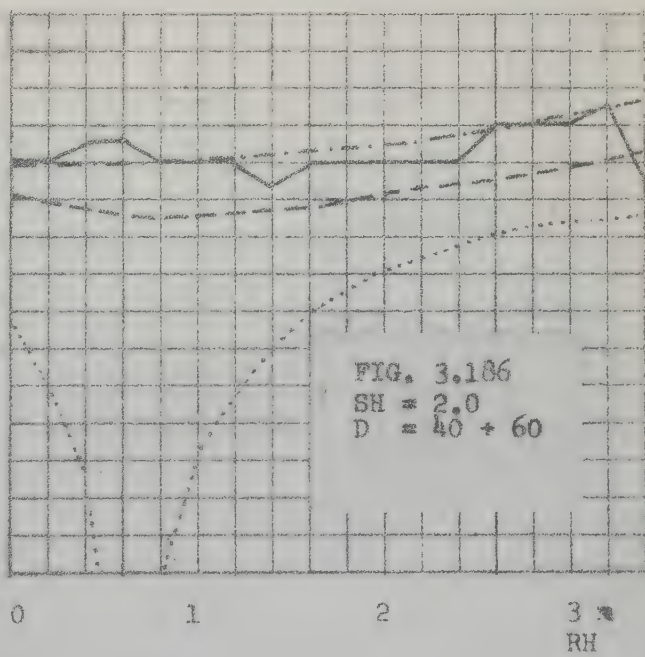
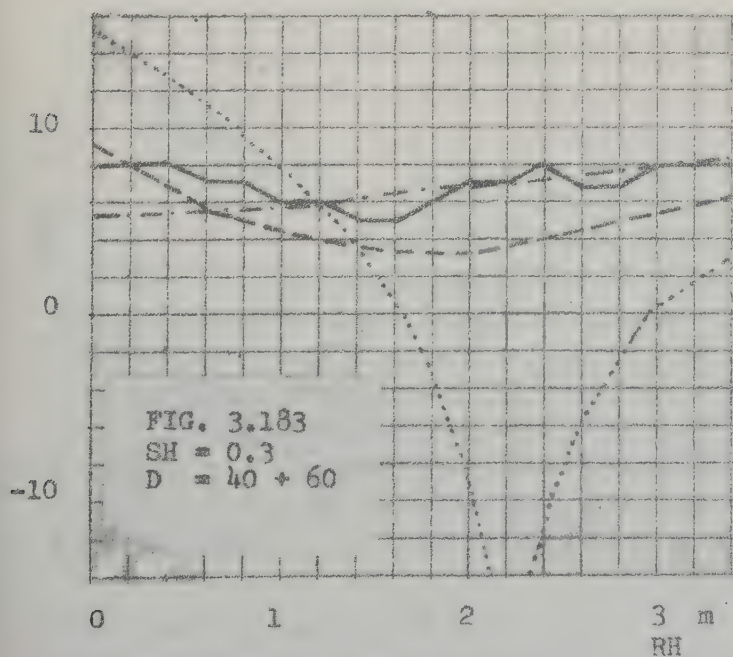
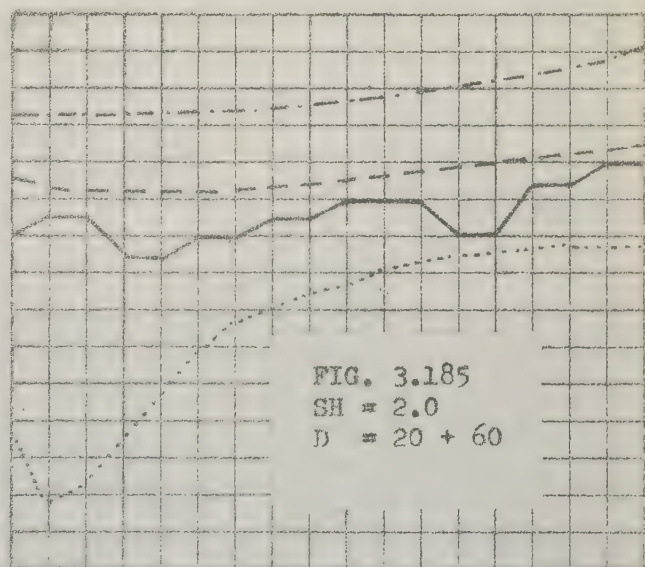
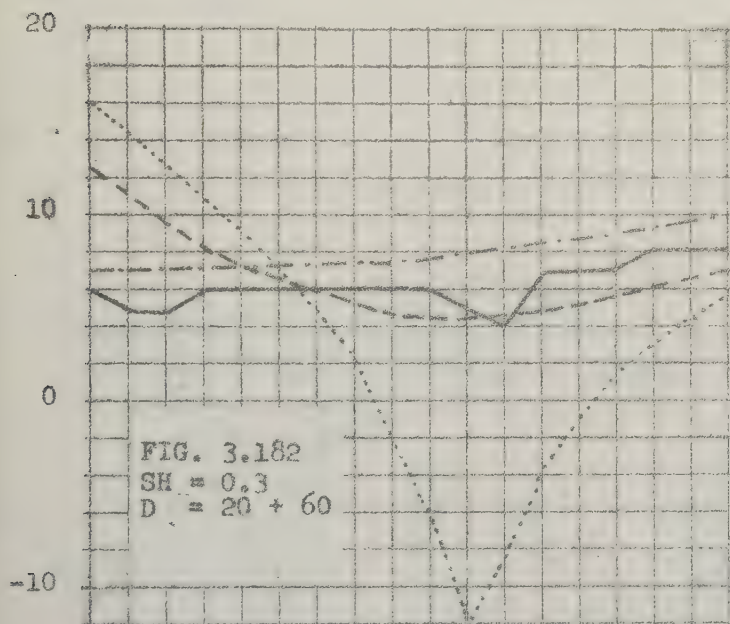
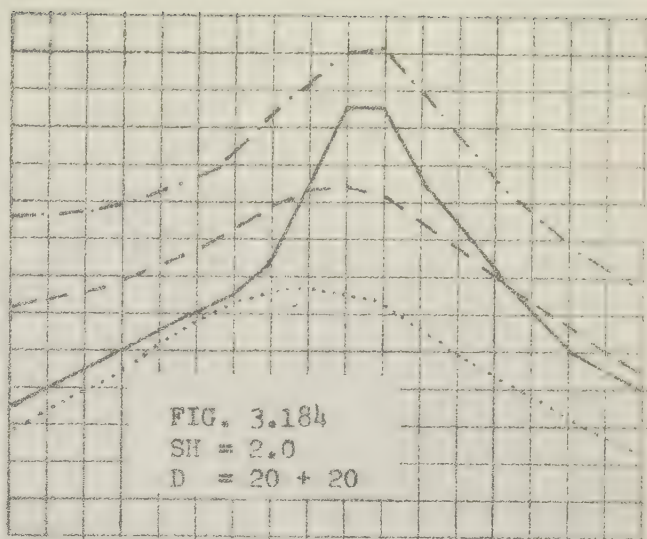
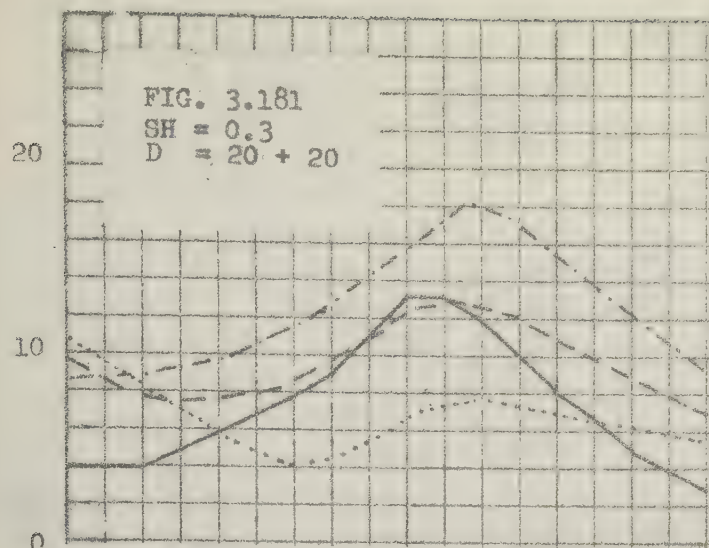
125 Hz

— measured

- - -  $\underline{u} = 0.005 - i \cdot 0.005$ - . -  $\underline{u} = 0.000 - i \cdot 0.000$







FIGS. 3.181 - 3.186. 250 Hz

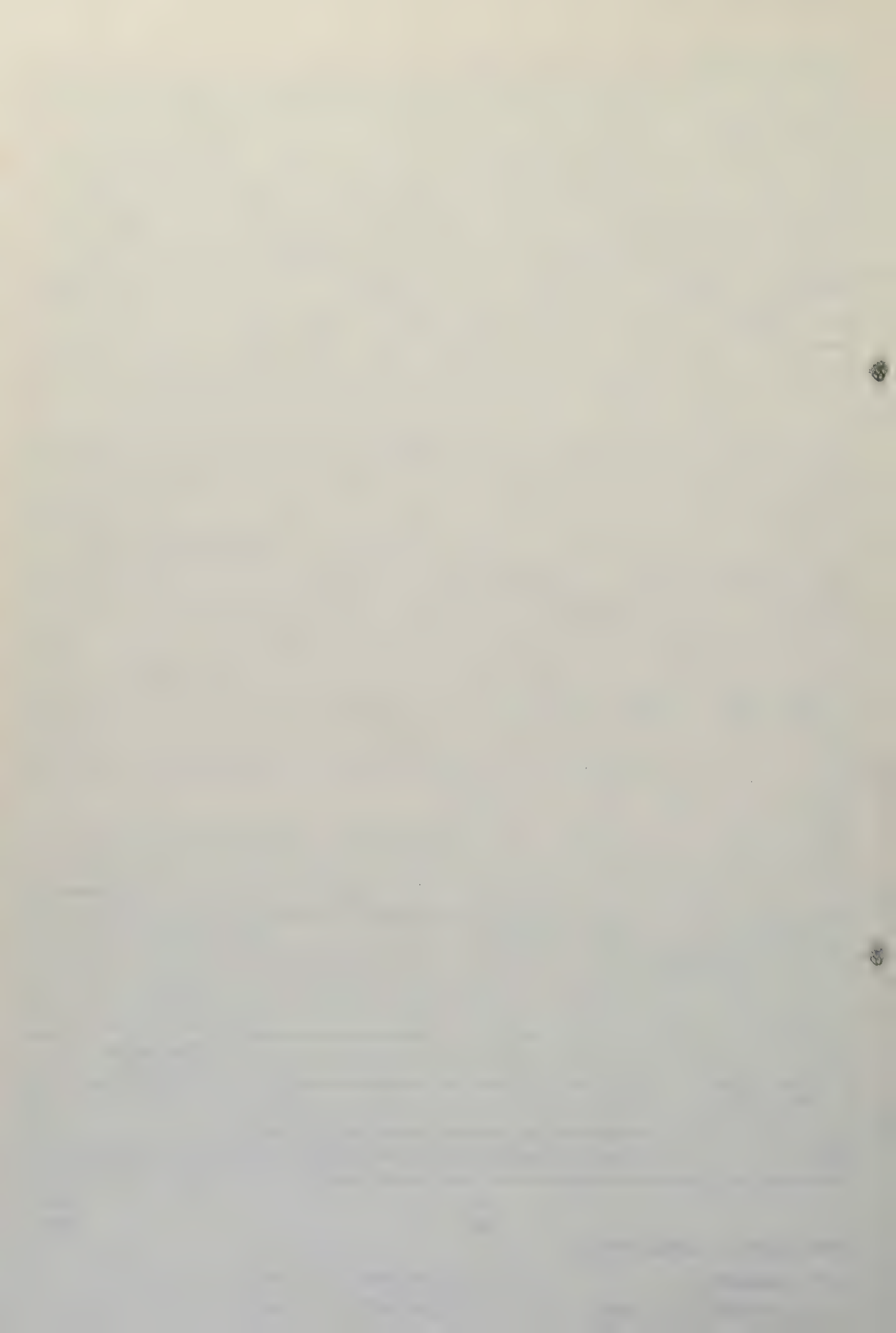
— measured

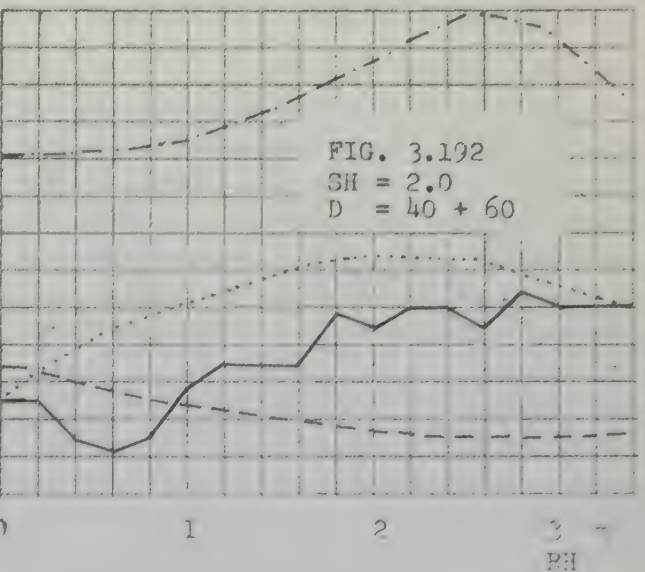
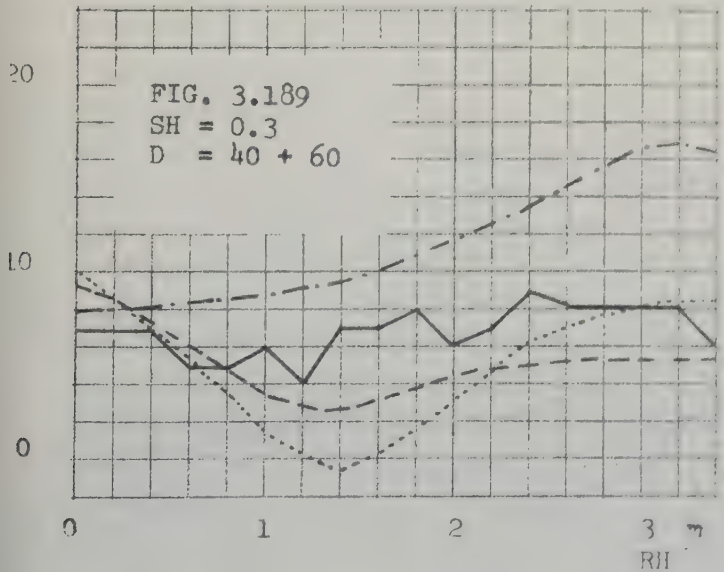
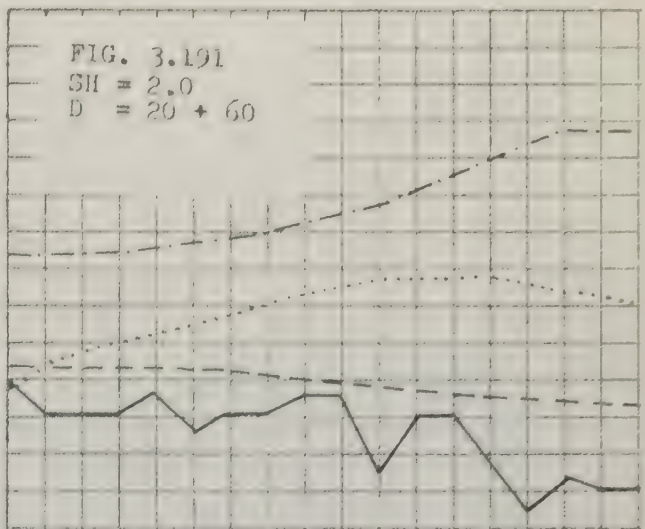
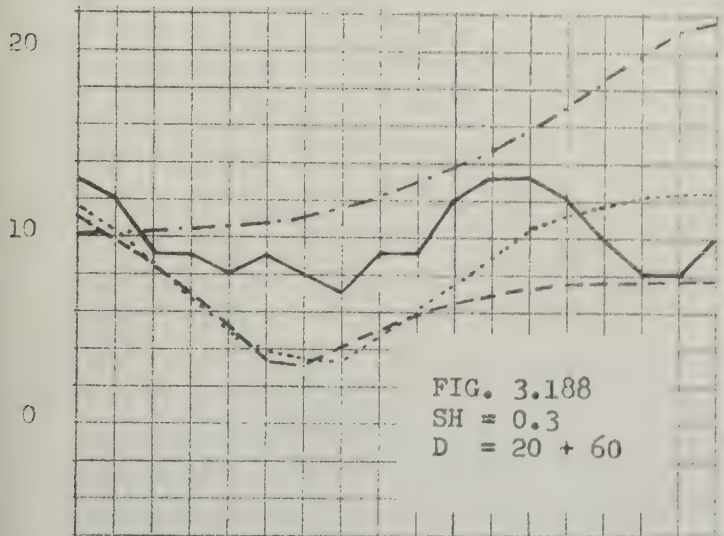
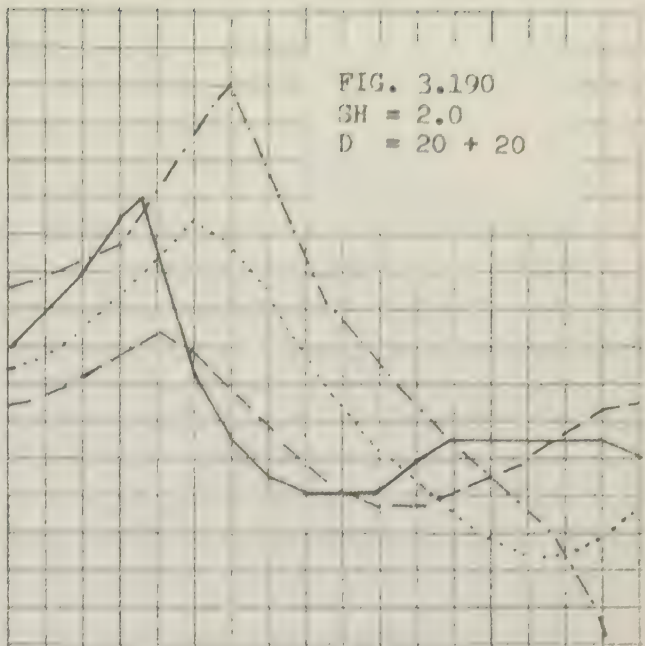
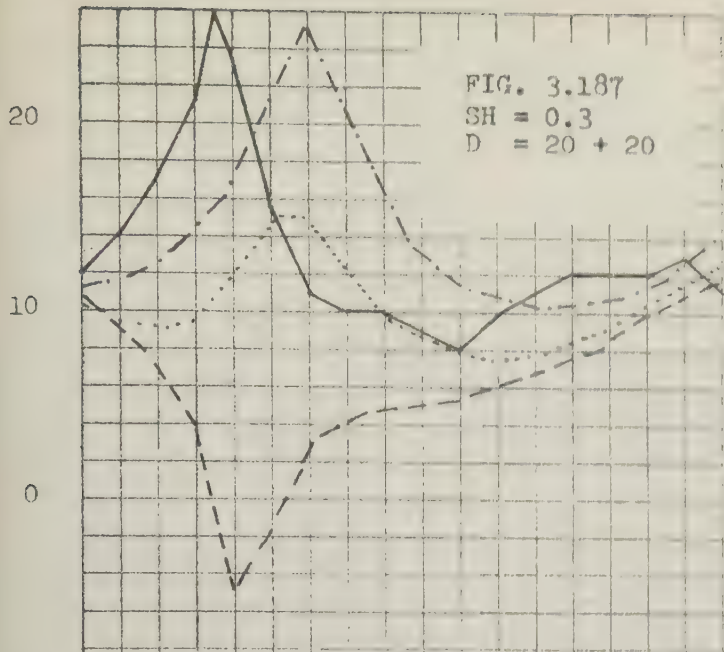
---  $\underline{u} = 0.03 - i \cdot 0.02$

...  $\underline{u} = 0.05 - i \cdot 0.05$

- - -  $\underline{u} = 0.00 - i \cdot 0.00$





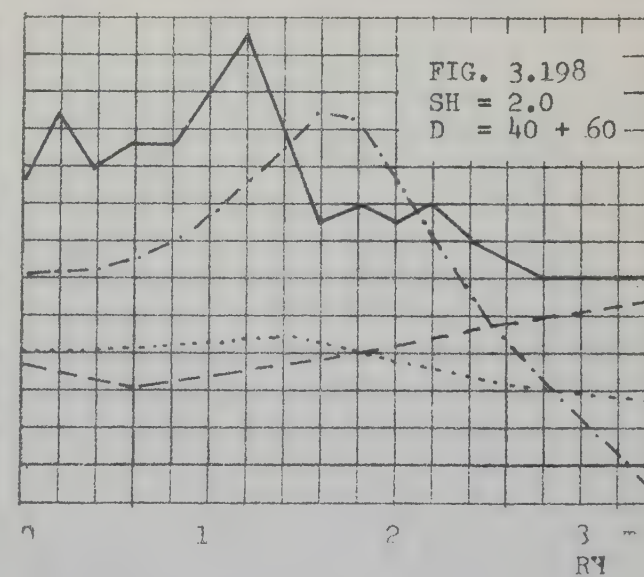
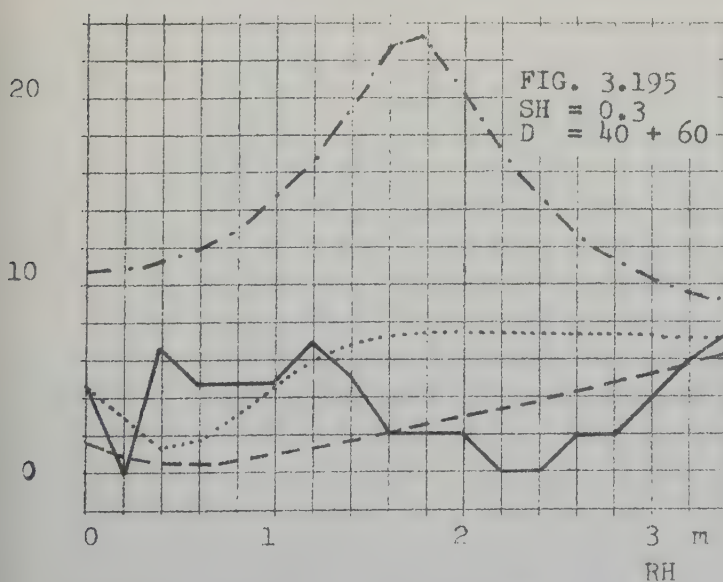
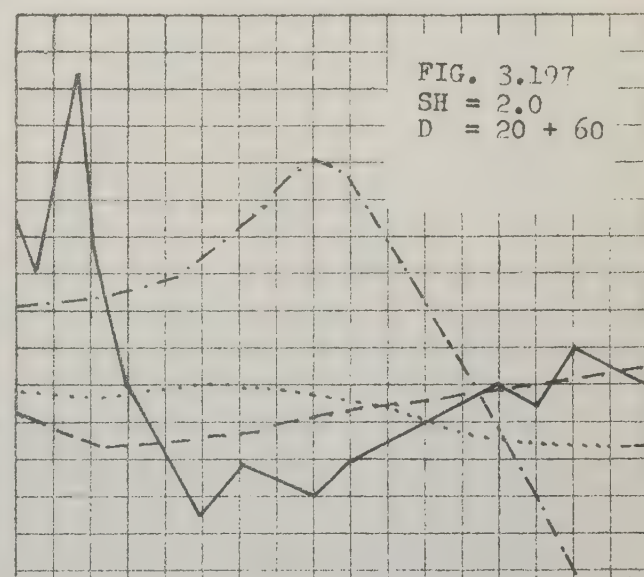
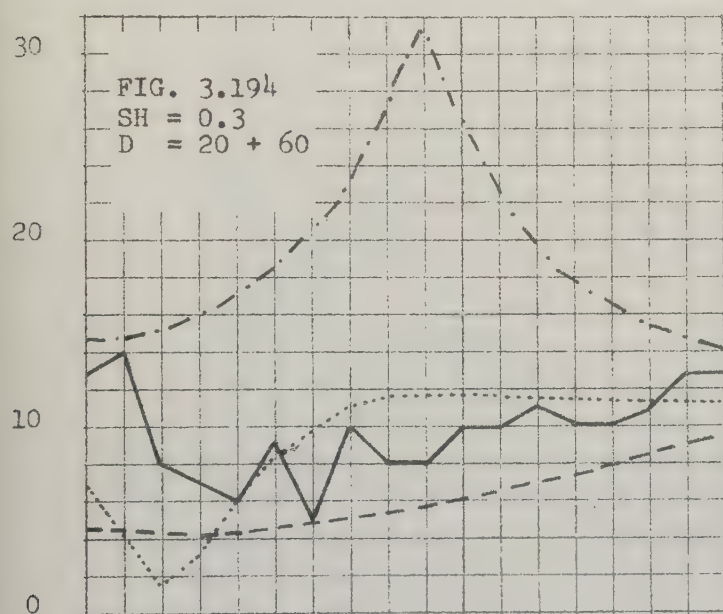
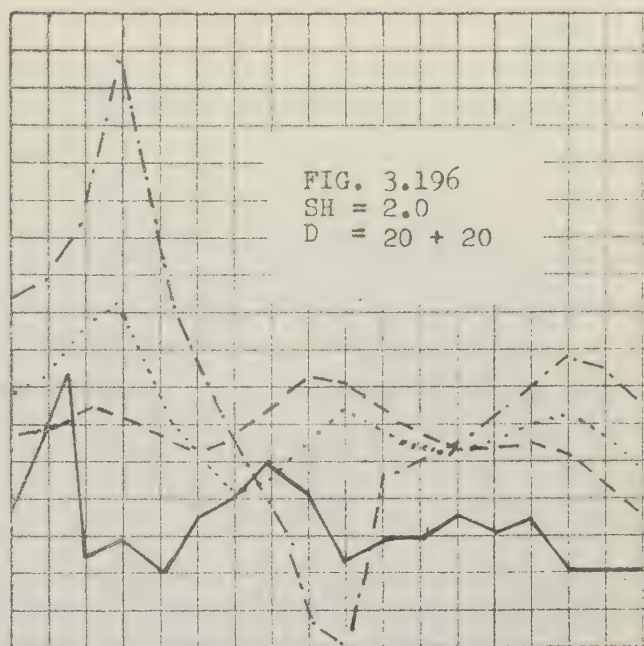
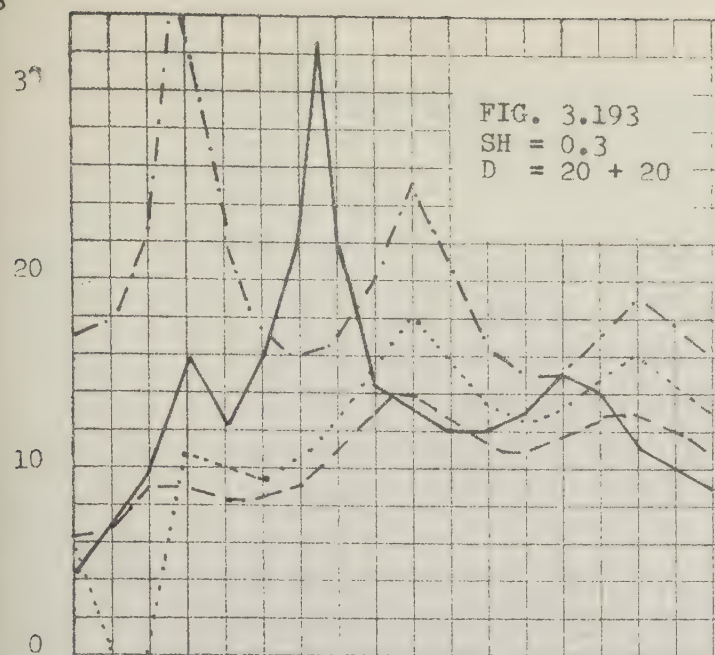


FIGS. 3.187 - 3.192. 500 Hz

— measured  
- - -  $\underline{u} = 0.06 - i \cdot 0.06$   
...  $\underline{u} = 0.03 - i \cdot 0.02$   
- . -  $\underline{u} = 0.00 - i \cdot 0.00$



1B



FIGS. 3.193 - 3.198. 1000 Hz

— measured

- - -  $\underline{u} = 0.10 - i \cdot 0.05$ ...  $\underline{u} = 0.05 - i \cdot 0.02$ - . -  $\underline{u} = 0.00 - i \cdot 0.00$

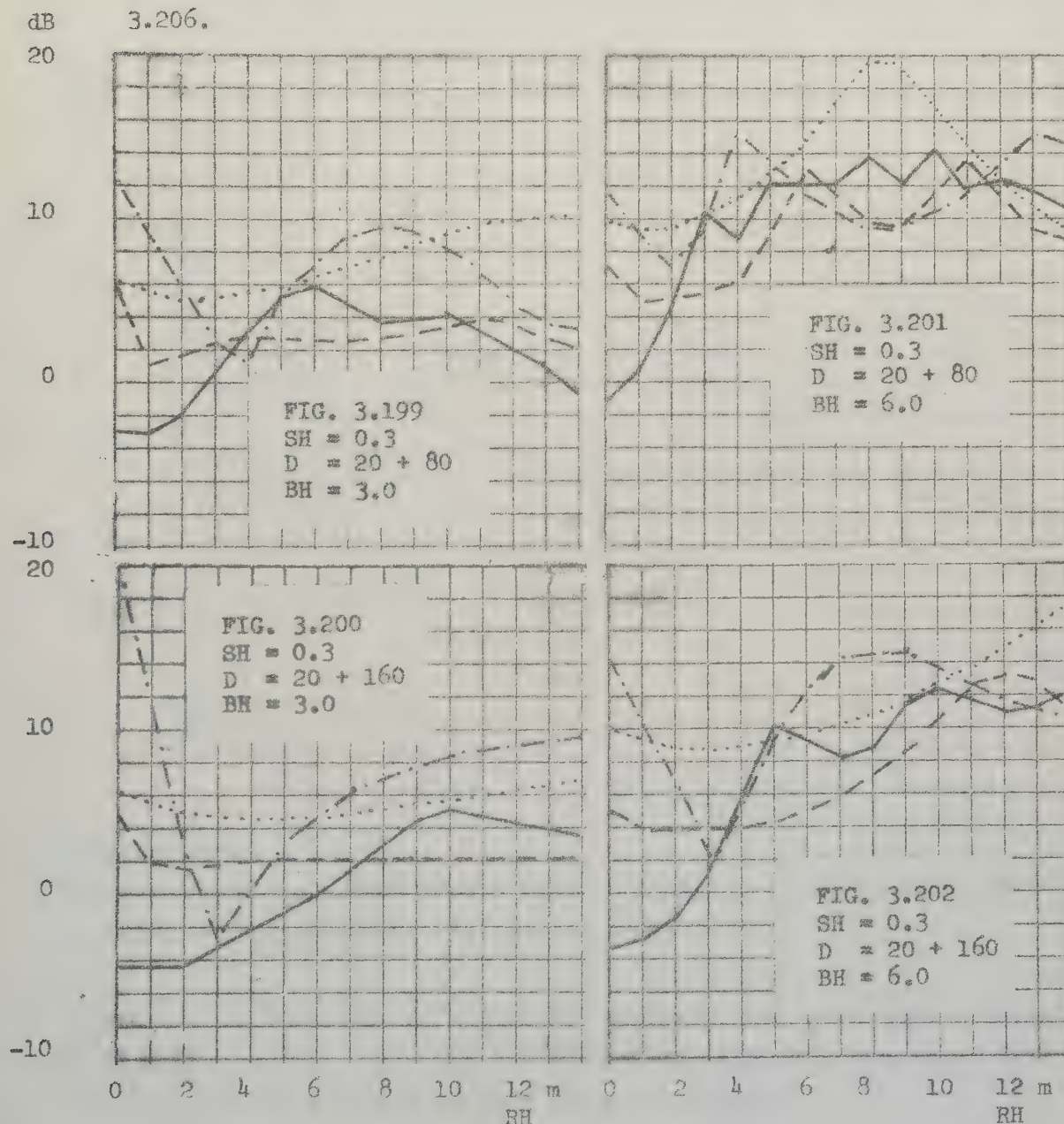






### 3.6.2. SOME CALCULATED EXAMPLES WITH INFINITE IMPEDANCE ON THE SOURCE SIDE

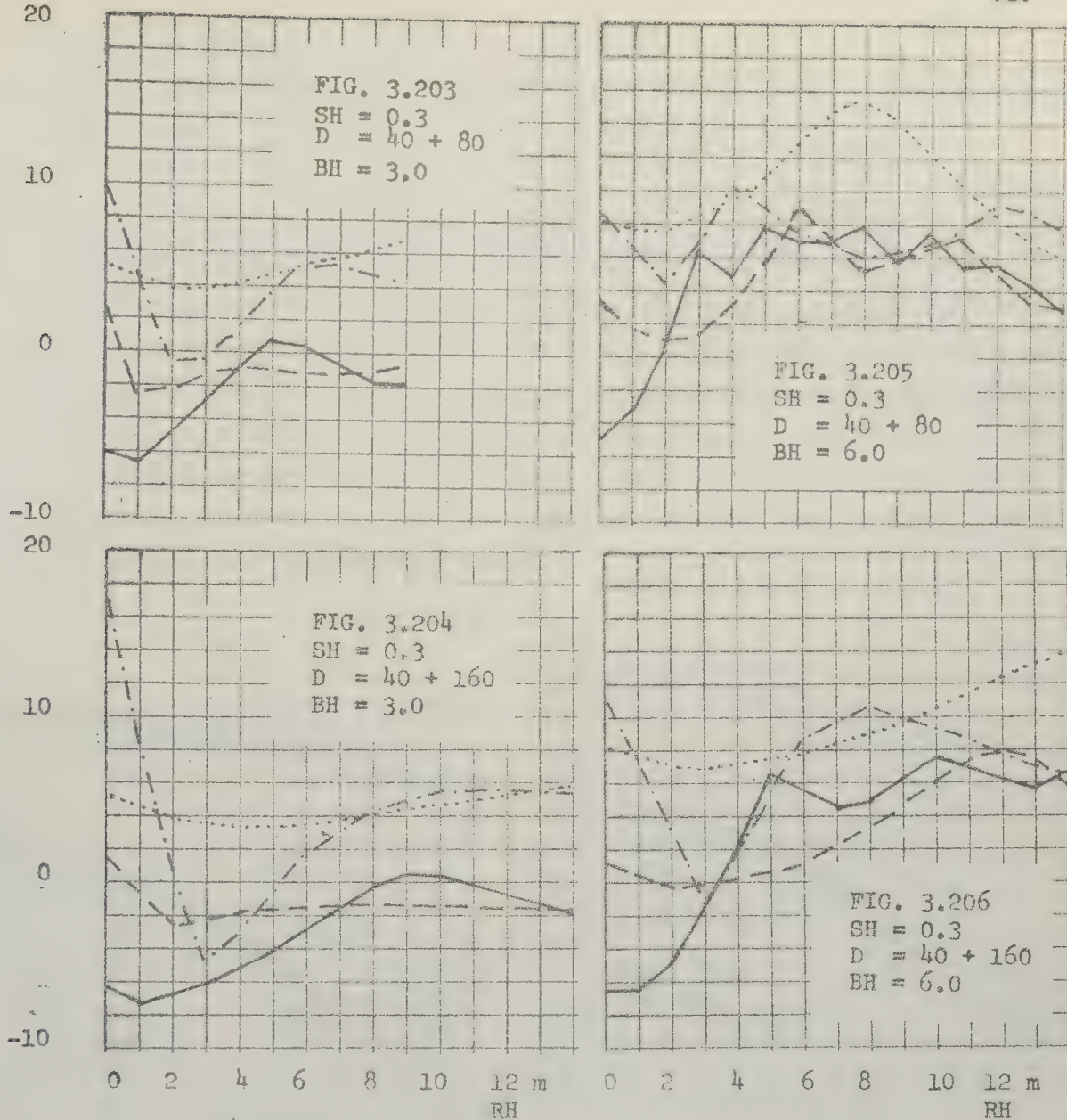
With barrier the ground surface is assumed to have an infinite impedance on the source side and a finite one on the receiver side. Eq.(2.141) has been used for the computations. Without barrier the reflections are supposed to take place on that part of the ground surface which has got a finite impedance. In this case the computations have been carried out in the same way as in Section 3.2.1. The results appear in Figs. 3.199 - 3.206.



FIGS. 3.199 - 3.202

— 1000 Hz,  $\underline{v} = 0.10 - i \cdot 0.05$     - - - 500 Hz,  $\underline{v} = 0.06 - i \cdot 0.06$   
 - . - 250 Hz,  $\underline{v} = 0.03 - i \cdot 0.02$     . . . 125 Hz,  $\underline{v} = 0.005 - i \cdot 0.005$





FIGS. 3.203 - 3.204

— 1000 Hz,  $\underline{u} = 0.10 - i \cdot 0.05$     - - - 500 Hz,  $\underline{u} = 0.06 - i \cdot 0.06$   
- . - 250 Hz,  $\underline{u} = 0.03 - i \cdot 0.02$     . . . 125 Hz,  $\underline{u} = 0.005 - i \cdot 0.005$

#### Some comments:

The frequency dependence of the insertion loss is of the same character as the one that was discussed in the last section.

The insertion loss does as a rule not decrease when the receiver height increases. This depends on the fact that the ground attenuation is decreased less by the barrier when the receiver is high above the ground.

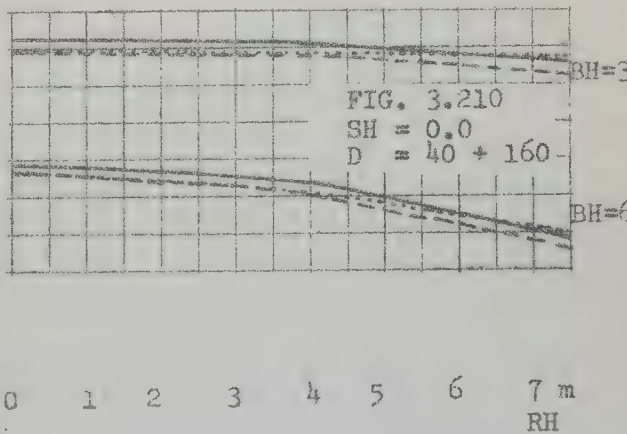
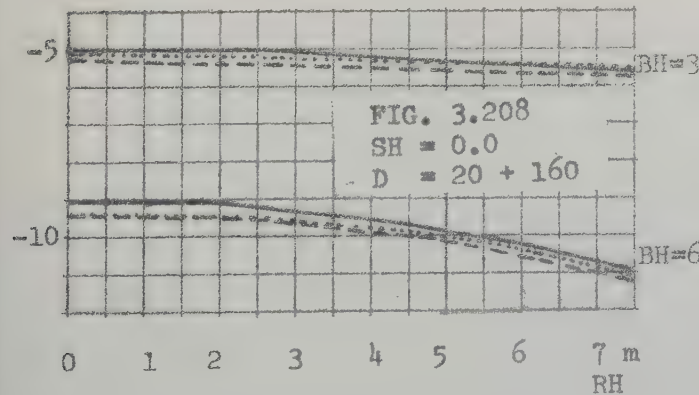
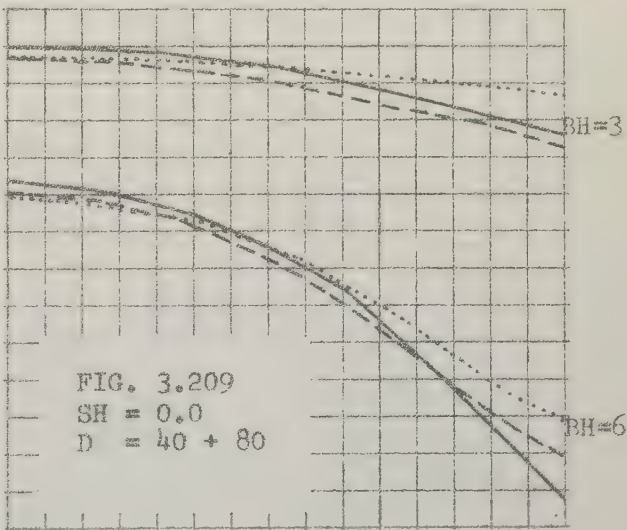
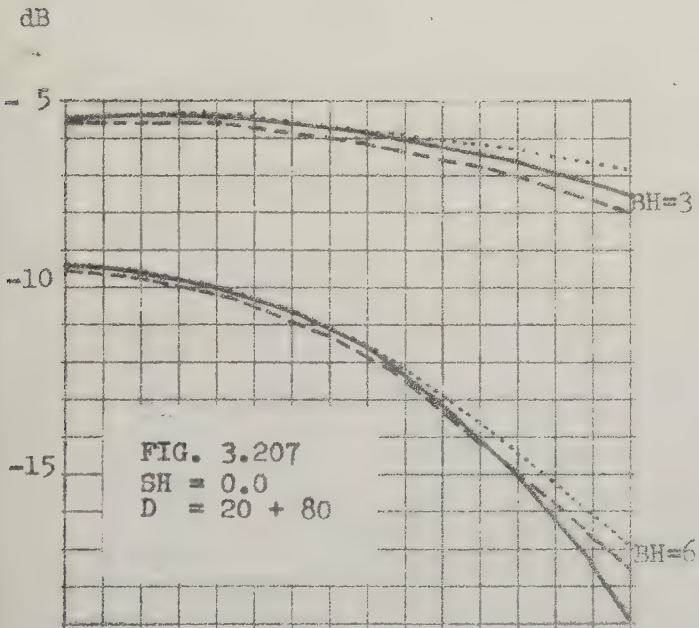
It should be observed that the improvement due to a barrier is rather modest.





3.6.3. SOME CALCULATED EXAMPLES FOR 125 Hz WITH INFINITE IMPEDANCE ON BOTH SIDES

At 125 Hz the ground impedance is likely to be almost infinite. As will be discussed in Section 4 this frequency is also important in many practical applications. Thus the computations have been carried out for 125 Hz only. The approximate solution according to Eq. (2.142) has been used and it is compared with the "exact" solution. As practical measurements generally are performed in octave bands the computations have also been carried out for the 125 Hz octave band by adding the 3 1/3 octave bands. The reference sound pressure level in Figs. 3.207 - 3.210 is the free level + 6 dB, that is the insertion loss with reversed sign.



FIGS. 3.207 - 3.210. 125 Hz

— Midband, exact                      - - - Octave band, exact  
... Octave band, approximate





Some comments:

The difference between the octave band and the midband frequency is insignificant in those cases which are dealt with. The agreement between the approximate and the "exact" solution is also satisfying. It is however evident from the analysis in Section 2.3.2.4 that the agreement will be worse close to the barrier.

The barriers are considerably more effective when the receiver is situated high above the ground. This is not surprising. Compare for example with Fig. 3.59.



#### 4. CONCLUSIONS AND PRACTICAL APPLICATIONS

A prerequisite for all calculations concerning the propagation of sound over ground is that the ground admittance is known. If this is the case the ground attenuation and the reduction by a barrier on the ground can be calculated with a high degree of accuracy by using the methods of this paper, providing of course that the ground surface is reasonably level and uniform.

Unfortunately the acoustic properties of the ground are little known. As long as this lack of knowledge is remaining the exact theories must be applied with great care. This is the more evident as the results of Section 3.2.1 indicate that at some critical frequencies even small admittance changes will affect the propagation pattern appreciably. As different ground areas are reflecting the incident sound waves at different receiver and source heights it is also possible that more than one value of the ground admittance must be used when calculating the height dependence of the sound pressure level. Besides there will probably also be seasonal variations in the ground admittance.

With all these uncertain factors, to which the direct influence of different meteorologic conditions must be added, it seems reasonable to make use of the approximate solutions to the largest possible extent. For practical applications to the propagation of traffic noise some of the observed qualitative results can be very useful.

It is beyond doubt that the propagation of sound over ground is strongly dependent on the frequency. This means that those frequencies which dominate the noise spectrum at one point not necessarily will dominate at another one. Traffic noise, which is generally measured in dB(A), is close to the source, after A-weighting, often dominated by frequencies around 1000 Hz. As low frequencies are attenuated less than high ones they will contribute more to the dB(A)-level at distant points than at points close to the road. For traffic noise indoors this shift towards lower frequencies will be further accentuated as the transmission loss of windows generally increases with the frequency. Hence it should be a reasonable approximation just to consider the 125 Hz octave band, which after A-weighting often is the





dominating low-frequency band. At this frequency the ground impedance can be assumed to be infinite and thus all computations are considerably simplified. When there is a barrier Eq. (2.143) can be used. The advantage of this approximate solution is that the phase factor of the function  $M$  is eliminated.

When the low-frequency model is valid it is obvious that the prevalent notion that the sound pressure level increases with the height of the observer is false.

If the barrier is situated close to a road Eq. (2.140) is valid. The only factor in this equation which cannot be calculated from geometrical considerations only is  $Q_{12}$ , which is a function of the ground admittance. If  $r \gg r_0$  and we just want to know the insertion loss of a projected barrier  $Q_{12}$  can be calculated from Eq. (2.24) if sound pressure measurements are carried out with the sound source at a place corresponding to the top of the barrier. Thus the ground admittance must not be known in this case.

The calculated examples in Section 3.6.2 indicate that the insertion loss of a barrier is often small and even negative. This depends on the fact that a barrier generally decreases the proper ground attenuation. This effect is most striking when the ground attenuation without the barrier is high. In extreme cases the decrease in ground attenuation may counteract the sound reduction due to diffraction to such an extent that the insertion loss of the barrier becomes negative. The conclusion is that an acoustic barrier is most effective at places where the ground attenuation is low. This is for example the case at low frequencies or, when the sound source is situated more than about 1 m above the ground level.



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